12 Using a graphic display calculator

CHAPTER OBJECTIVES:

This chapter shows you how to use your graphic display calculator (GDC) to solve the different types of problems that you will meet in your course. You should not work through the whole of the chapter – it is simply here for reference purposes. When you are working on problems in the mathematical chapters, you can refer to this chapter for extra help with your GDC if you need it.

Instructions for the TI-84 Plus calculator

Use this list to help you to find the topic you need

Table of contents

Before you start

You should be familiar with:

- Important keys on the keyboard: ON 2nd DEL CLEAR Y= X,T,Θ,n ENTER GRAPH
- The home screen
- Changing window settings in the graph screen
- Using zoom tools in the graph screen
- Using trace in the graph screen

For a reminder of how to perform the basic operations have a look at your GDC manual.

1 Number and algebra 1

Simultaneous and quadratic equations

1.1 Solving simultaneous linear equations

When solving simultaneous equations in an examination, you do not need to show any method of solution. You should simply write out the equations in the correct form and then give the solutions. The calculator will do all the working for you.

You will need to have the App PlySmlt2 installed on your GDC. This App is permitted by IBO in your examination.

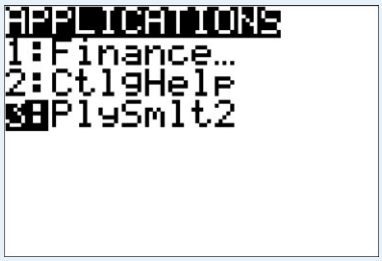
Example 1

Solve the equations:

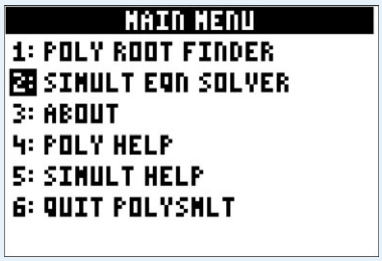
$2x + y = 10$

$x - y = 2$

Press APPS. You will see the dialog box as shown on the right. Choose the App PlySmlt2 and press ENTER.

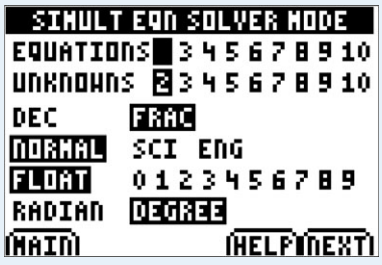


From the main menu, choose 2: SIMULT EQN SOLVER and press ENTER.



The defaults are to solve two equations in two unknowns.

Note: This is how you will use the linear equation solver in your examinations. In your project, you might want to solve a more complicated system with more equations and more variables.



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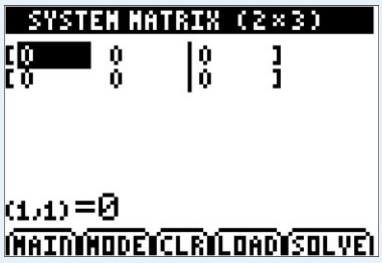
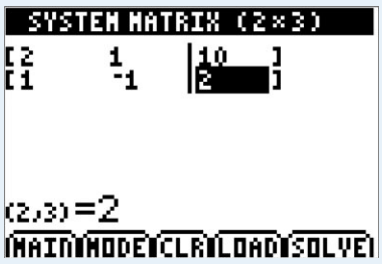
Press F5 and you will see the template on the right.

Type the coefficients from two equations into the template, pressing ENTER after each number.

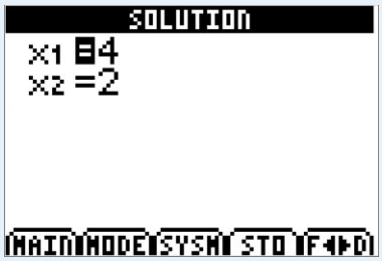
The equations must be in the correct order.



Press F5 and the calculator will solve the equations, giving the solutions as x_1 and x_2.



The solutions are $x = 4$, $y = 2$.



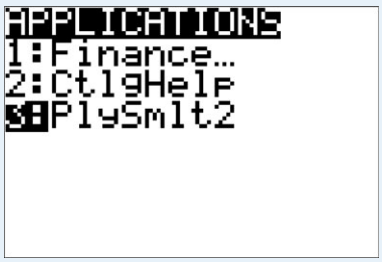
1.2 Solving quadratic equations

When solving quadratic equations in an examination, you do not need to show any method of solution. You should simply write out the equations in the correct form and then give the solutions. The calculator will do all the working for you.

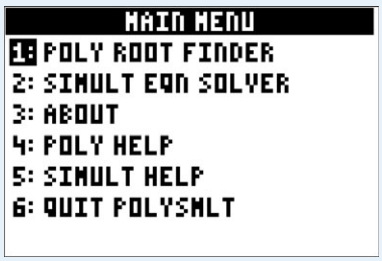
Example 2

Solve $3x^2 - 4x - 2 = 0$

Press APPS. You will see the dialog box as shown on the right. Choose the App PlySmlt2 and press ENTER.



From the main menu, choose 1: POLY ROOT FINDER and press ENTER.



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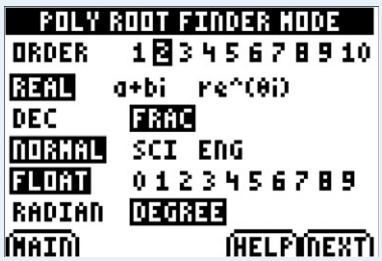
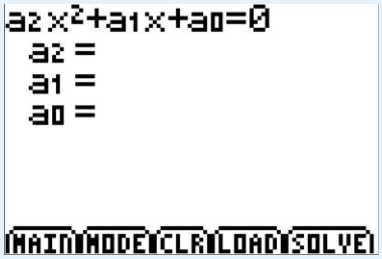
The defaults are to solve an equation of order 2 (a quadratic equation) with real roots. You do not need to change anything.



Another dialog box opens for you to enter the equation.

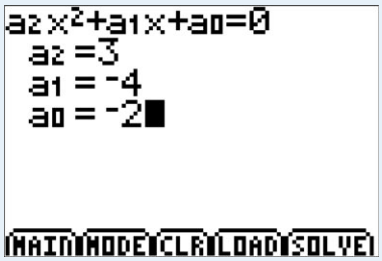
The general form of the quadratic equation is $a_2x^2 + a_1x + a_0 = 0$, so we enter the coefficients in a_2, a_1 and a_0.



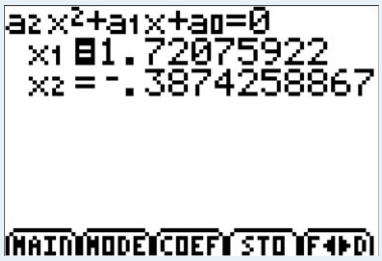
Here $a2 = 3$, $a1 = -4$ and $a0 = -2$. Be sure to use the (-) key to enter the negative values.

Press ENTER after each value.

Press F5 and the calculator will find the roots of the equation.



The solutions are $x = -0.387$ or $x = 1.72$ (3 sf).



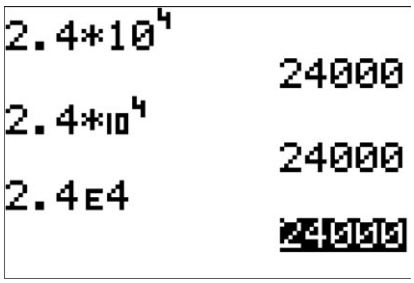
Standard form and significant figures

1.3 Standard form

Numbers written in standard form are in the form $a \times 10^n$, where $1 \leq a < 10$ and $n \in \mathbb{Z}$.

There are three different ways of entering a number in standard form. For example, to enter 2.4×10^4 press the keys

2 . 4 × 1 0 ^ 4 or
2 . 4 × 10^ 4 or
2 . 4 EE 4



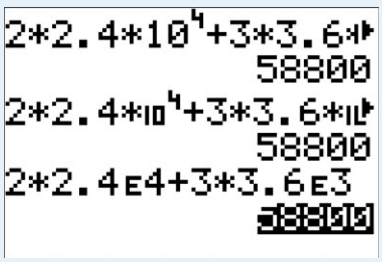
Example 3

Given that $x = 2.4 \times 10^4$ and $y = 3.6 \times 10^3$, find the value of

a $2x + 3y$ **b** xy^2

Enter the values using one of the three methods previously highlighted. In normal mode, the calculator will display the result either as a normal number or, if it is a large number, in standard form.

Write your answer in standard form. For 3.11E11, write 3.11×10^{11}.



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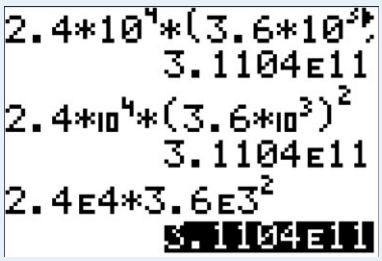
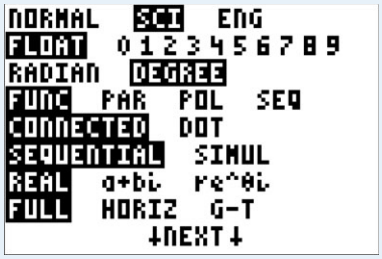
After you enter an exponent, you will need to press ▸ to return to the base line of the calculation.

When you enter a number using either ^ or 2nd 10^x, you should use brackets before you square the number.



To change the answer to standard form, press MODE
Choose 'SCI' for the Exponential Format.
Press 2nd QUIT to return to the calculation page.

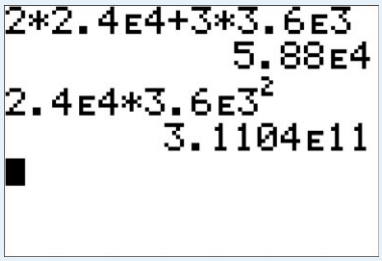
Note: Remember to return the settings back to normal when you have finished.



All results are now given in standard form:

a 5.88×10^4

b 3.11×10^{11}



1.4 Significant figures

For Mathematical Studies, answers to questions should be rounded to 3 significant figures (3 sf), unless told otherwise.

Your GDC is not much help as it will only round to a set number of decimal places (dp).

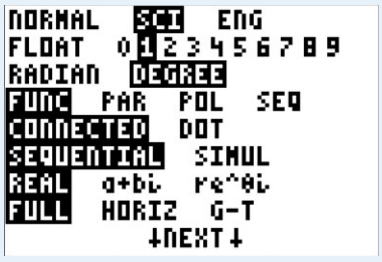
Example 4

Do these calculations. Give each of your answers correct to 3 significant figures (3 sf).

a $4 \times \pi$ **b** 3.629×2.76 **c** 123×12

To change the answer to 1 decimal place, press MODE.
Choose '1' for the number of decimal places.
Press 2nd QUIT to return to the calculation page.

Note: Remember to return the settings back to normal when you have finished.

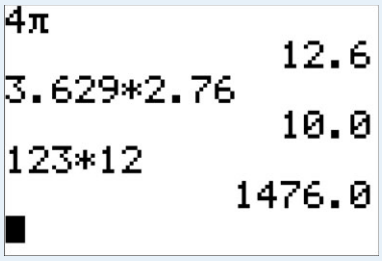


All results are now given in rounded form:

a 12.6 is correct to 1 dp. This is the same as 3 sf for this example.

b 10.0 is correct to 1 dp. This is the same as 3 sf for this example.

c 1476 is 1480 to 3 sf. Since the rounding is before the decimal point, the GDC will not round this answer.



2 Descriptive statistics

You can use your GDC to draw charts to represent data and to calculate basic statistics such as mean, median, etc. Before you do this you need to enter the data in a list.

Entering data

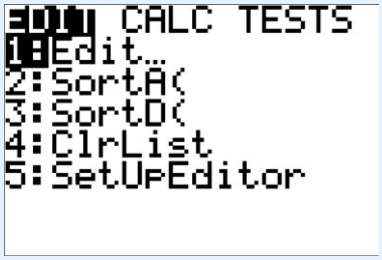
There are two ways of entering data: as a list or as a frequency table.

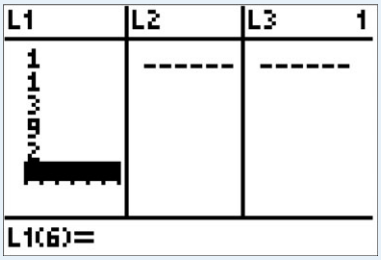
2.1 Entering lists of data

Example 5

Enter the data in the list: 1, 1, 3, 9, 2.

Press STAT 1: Edit and press ENTER.
Type the numbers in the first column (L1).
Press ENTER or ▼ after each number to move down to the next cell.
L1 will be used later when you want to make a chart or to do some calculations with this data. You can use columns from L1 to L6 to enter the list.





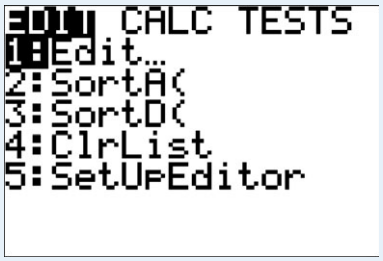
2.2 Entering data from a frequency table

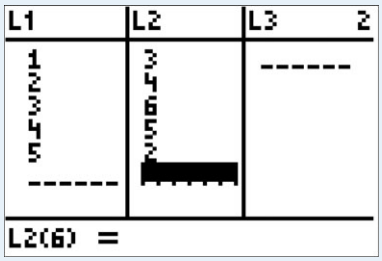
Example 6

Enter the data in the table:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

Press STAT | 1:Edit and press ENTER.
Type the numbers in the first column (L1) and the frequencies in the second column (L2).
Press ENTER or ▼ after each number to move down to the next cell.
Press ▶ to move to the next column.
L1 and L2 will be used later when you want to make a chart or to do some calculations with this data. You can use columns from L1 to L6 to enter the lists.





Drawing charts

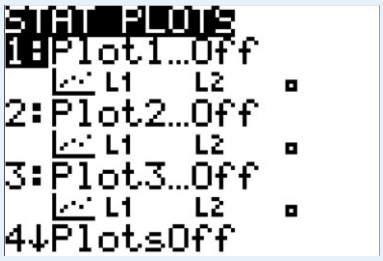
Charts can be drawn from a list or from a frequency table.

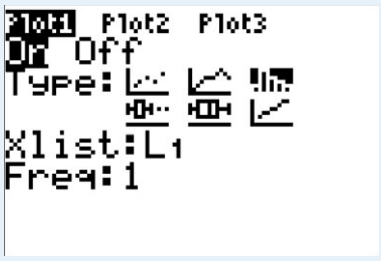
2.3 Drawing a frequency histogram from a list

Example 7

Draw a frequency histogram for this data: 1, 1, 3, 9, 2.

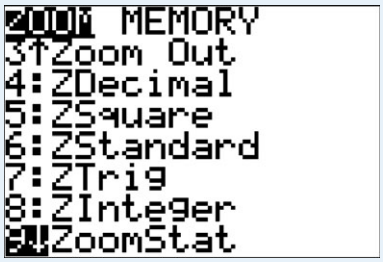
Enter the data in L1 (see Example 5).
Press 2nd STAT PLOT and ENTER to select Plot1.
Select On, choose the histogram option and leave XList as L1 and Freq as 1.

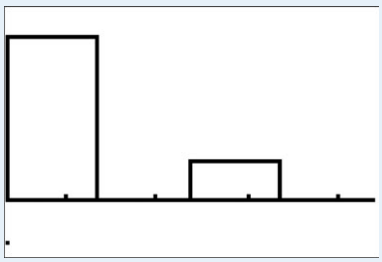




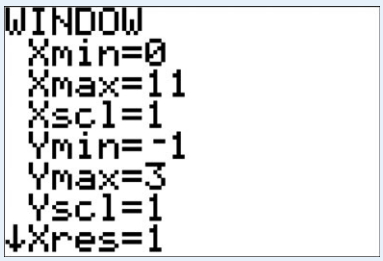
Press ZOOM | 9:Stat.
The automatic scales do not usually give the best display of the histogram. You will need to change the default values.

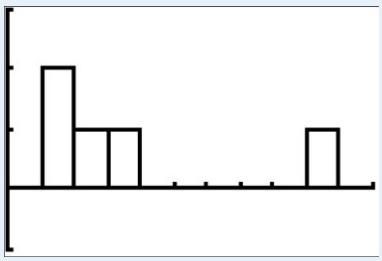
You may need to delete any function graphs. Y=





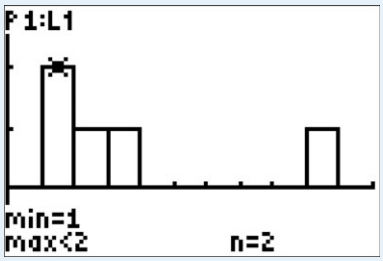
Press WINDOW and choose options as shown.
Xmin and Xmax should include the range of the data.
Ymin and Ymax should include the maximum frequency and should go below zero.
Xscl will define the width of the bars.





Press TRACE.
Use the ▸ key to move to each of the bars and display their value and frequency.

You should now see a frequency histogram for the data in the list 1, 1, 3, 9, 2.



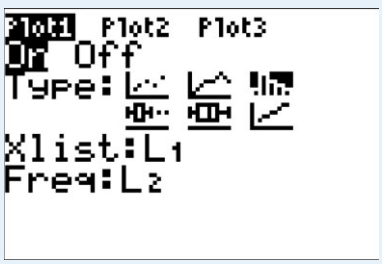
2.4 Drawing a frequency histogram from a frequency table

Example 8

Draw a frequency histogram for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

Enter the data in L1 and L2 (see Example 6).
Press 2nd STAT PLOT and ENTER to select Plot 1.
Select On, choose the histogram option and leave XList as L1 and Freq as L2.



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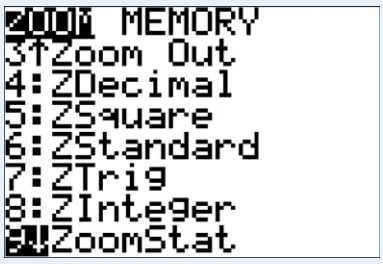
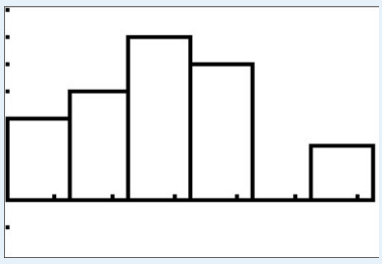
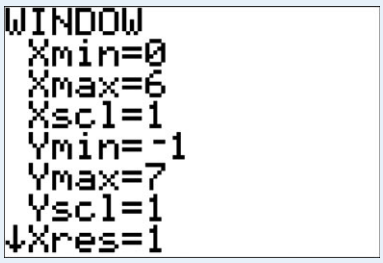
Press ZOOM | 9:Stat.
The automatic scales do not usually give the best display of the histogram. You will need to change the default values.

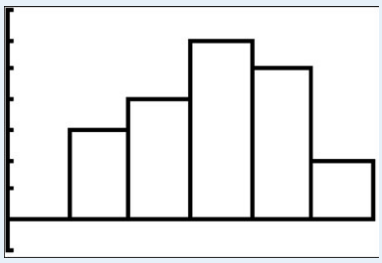
You may need to delete any function graphs. Y=





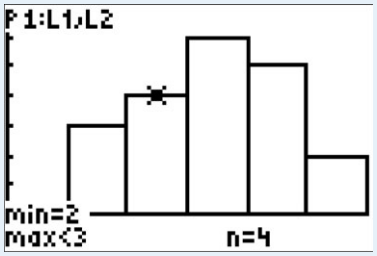
Press WINDOW and choose options as shown.
Xmin and Xmax should include the range of the data.
Ymin and Ymax should include the maximum frequency and should go below zero.
Xscl will define the width of the bars.





Press TRACE.
Use the ▸ key to move to each of the bars and display their value and frequency.

You should now see a frequency histogram for the data in the list 1, 1, 3, 9, 2.

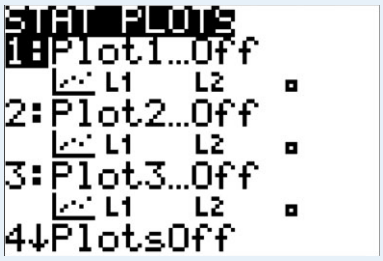


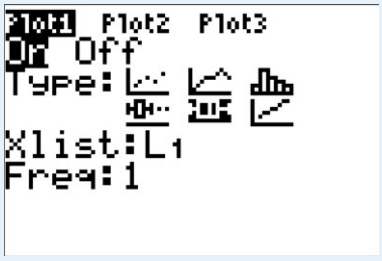
2.5 Drawing a box and whisker diagram from a list

Example 9

Draw a box and whisker diagram for this data:
1, 1, 3, 9, 2.

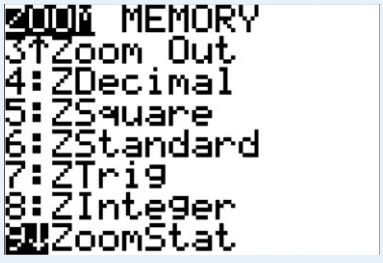
Enter the data in L1 (see Example 5).
Press 2nd STAT PLOT and ENTER to select Plot 1.
Select On, choose the box and whisker option and leave XList as L1 and Freq as 1.

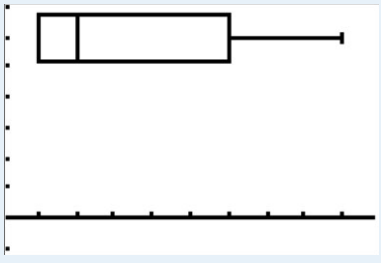




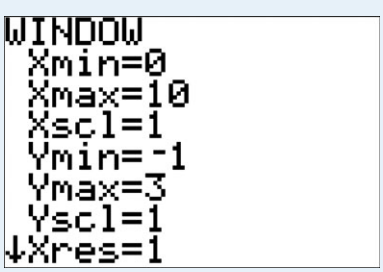
Press ZOOM | 9:Stat.
The automatic scales do not usually give the best display of the box and whisker diagram. You will need to change the default values.

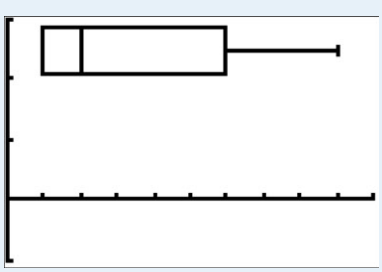
You may need to delete any function graphs. Y=





Press WINDOW and choose options as shown.
Xmin and Xmax should include the range of the data.
Ymin and Ymax do not affect the way in which the diagram is displayed.





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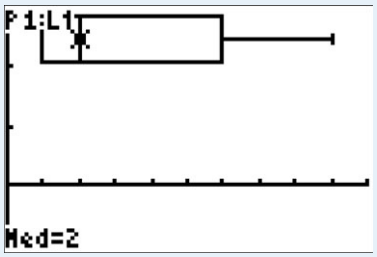
Press TRACE.
Use the ▸ key to move the cursor over the plot to see the quartiles, Q1 and Q3, the median and the maximum and minimum values.



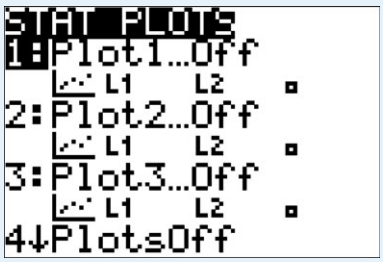
2.6 Drawing a box and whisker diagram from a frequency table

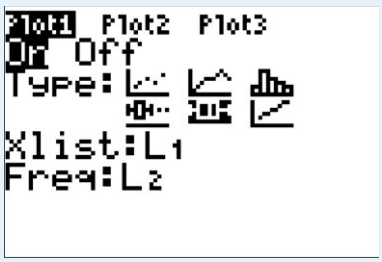
Example 10

Draw a box and whisker diagram for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

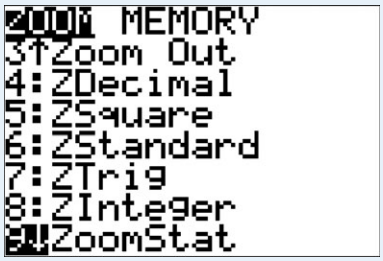
Enter the data in L1 and L2 (see Example 6).
Press 2nd STAT PLOT and ENTER to select Plot 1.
Select On, choose the box and whisker diagram option and leave XList as L1 and Freq as L2.

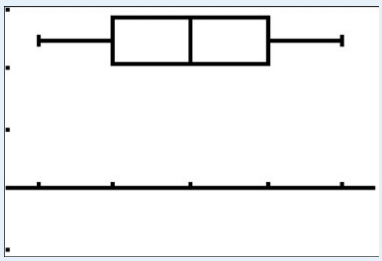




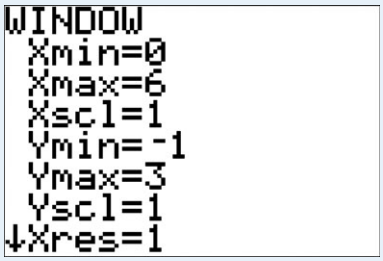
Press ZOOM | 9:Stat.
The automatic scales do not usually give the best display of the box and whisker diagram. You will need to change the default values.

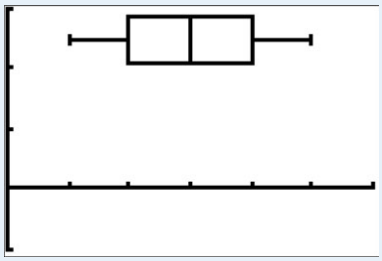
You may need to delete any function graphs. Y=



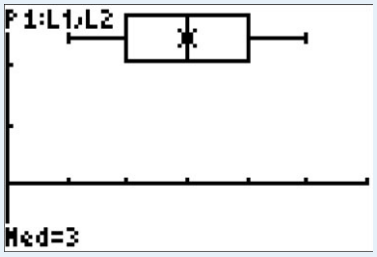


Press WINDOW and choose options as shown.
Xmin and Xmax should include the range of the data.
Ymin and Ymax do not affect the way in which the diagram is displayed.





Press TRACE.
Use the ▸ key to move the cursor over the plot to see the quartiles, Q1 and Q3, the median and the maximum and minimum values.



Calculating statistics

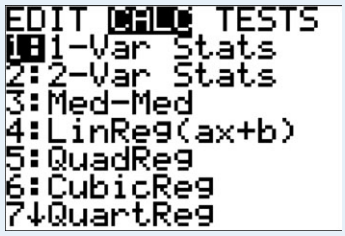
You can calculate statistics such as mean, median, etc. from a list, or from a frequency table.

2.7 Calculating statistics from a list

Example 11

Calculate the summary statistics for this data: 1, 1, 3, 9, 2

Enter the data in L1 (see Example 5).
Press STAT | CALC | 1:1-Var Stats.

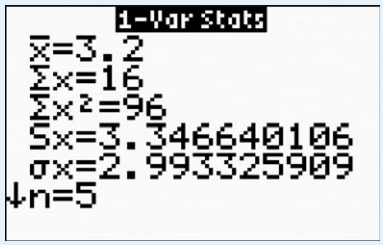


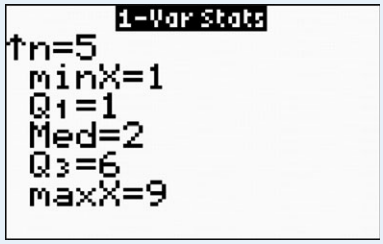
Fill in the wizard.
L1 is the default list,
For data in a list you should leave FreqList blank.
Select Calculate and press ENTER.

On clearing the home screen, scrolling up will bring up the command 1-Var Stats L1, which can then be pasted.

The information shown will not fit on a single screen.
You can scroll up and down to see it all.
The statistics calculated for the data are:

mean	$\bar{x}$
sum	Σx
sum of squares	Σx^2
sample standard deviation	s_x
population standard deviation	σ_x
number	n
minimum value	Min$X$
lower quartile	Q_1
median	Med
upper quartile	Q_3
maximum value	Max$X$





Note: You should always use the population standard deviation (σ_x) in Mathematical Studies.

2.8 Calculating statistics from a frequency table

Example 12

Calculate the summary statistics for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

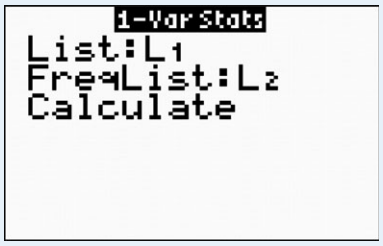
▶ Continued on next page

Enter the data in L1 and L2 (see Example 6).
Press STAT | CALC | 1:1-Var Stats.

Fill in the wizard.
L1 is the default list, type 2nd L2 in FreqList

Select Calculate and press ENTER.

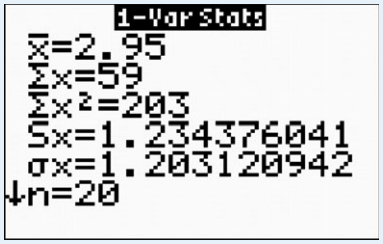
On clearing the home screen, scrolling up will bring up the command 1-Var Stats L1, L2 which can then be pasted.

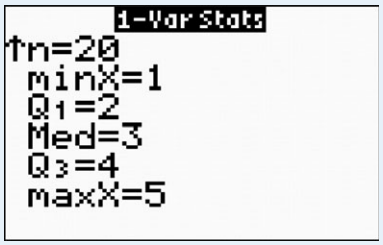


The information shown will not fit on a single screen. You can scroll up and down to see it all.
The statistics calculated for the data are:

mean	$\bar{x}$
sum	Σx
sum of squares	Σx^2
sample standard deviation	s_x
population standard deviation	σ_x
number	n
minimum value	min$X$
lower quartile	Q_1
median	Med
upper quartile	Q_3
maximum value	Max$X$

Note: You should always use the population standard deviation (σ_x) in Mathematical Studies.





2.9 Calculating the interquartile range

Example 13

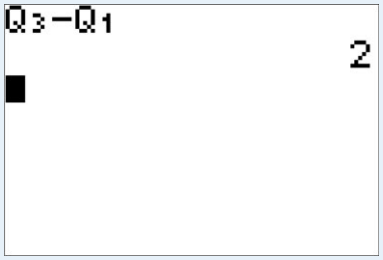
Calculate the interquartile range for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

The interquartile range is the difference between the upper and lower quartiles ($Q_3 - Q_1$).

First calculate the summary statistics for this data (see Example 12).

(**Note**: The values of the summary statistics are stored after One-Variable Statistics have been calculated and remain stored until the next time they are calculated.)
Press VARS | 5:Statistics... | PTS | 9:Q3 ENTER – VARS | 5:Statistics... | PTS | 7:Q1 ENTER
The calculator now displays the result:
Interquartile range = $Q_3 - Q_1 = 2$



2.10 Using statistics

The calculator stores the values you calculate in One-Variable Statistics so that you can access them in other calculations. These values are stored until you do another One-Variable Statistics calculation.

Example 14

Calculate the $\bar{x} + \sigma_x$ for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

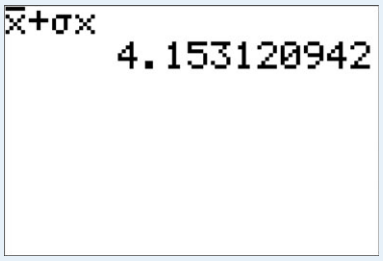
First calculate the summary statistics for this data (see Example 12).

(**Note**: The values of the summary statistics are stored after One-Variable Statistics have been calculated and remain stored until the next time they are calculated.)

Press VARS | 5:Statistics... | 2: $\bar{x}$ ENTER + VARS | 5:Statistics... 4:σx ENTER

The calculator now displays the result:

$\bar{x} + \sigma_x = 4.15$ (to 3sf)



3 Geometry and trigonometry

This section covers the display of and reading of information from graphs of linear functions.

Graphing

3.1 Graphing linear functions

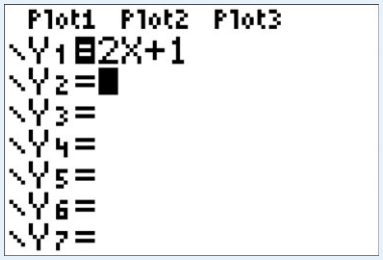
Example 15

Draw the graph of the function $y = 2x + 1$.

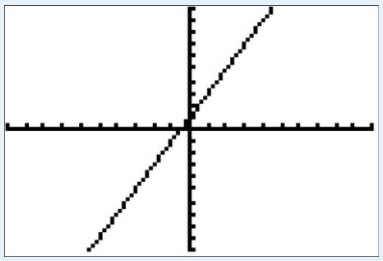
Press Y= to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

Type $2x + 1$ and press ENTER.

Press ZOOM | 6:ZStandard to use the default axes which are $-10 \le x \le 10$ and $-10 \le y \le 10$.



The graph of $y = 2x + 1$ is now displayed on the screen.



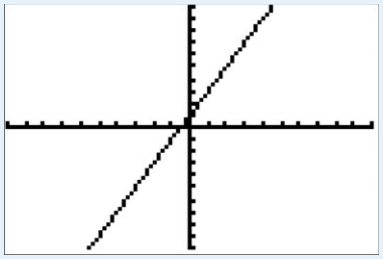
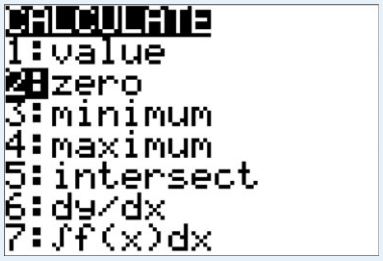
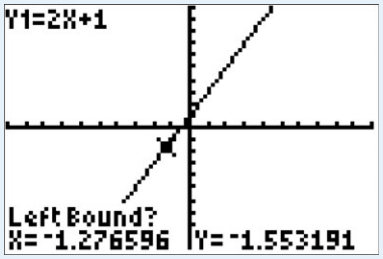
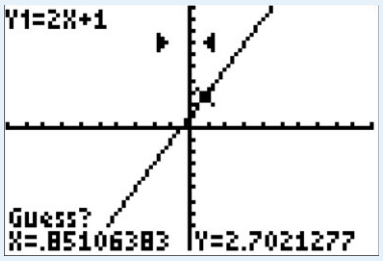
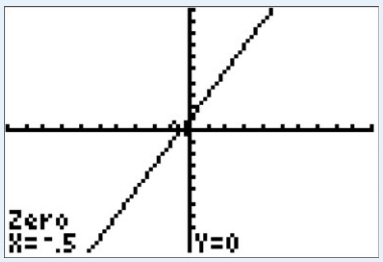
Finding information about the graph

The GDC can give you a lot of information about the graph of a function, such as the coordinates of points of interest and the gradient (slope).

3.2 Finding a zero

The x-intercept is known as a *zero* of the function.

Example 16

Find the zero of $y = 2x + 1$.	
Draw the graph of $y = 2x + 1$ as in Example 15.	
Press 2nd CALC | 2:Zero Press ENTER	
To find the zero you need to give the left and right bounds of a region that includes the zero. The calculator shows a point and asks you to set the left bound. Move the point using the ◀ and ▶ keys to choose a position to the left of the zero. Press ENTER.	
The calculator shows another point and asks you to set the right bound. Move the point using the ◀ and ▶ keys so that the region between the left and right bounds contains the zero. When the region contains the zero press ENTER.	
Press ENTER again to supply a guess for the value of the zero. The calculator displays the zero of the function $y = 2x + 1$ at the point $(-0.5, 0)$.	

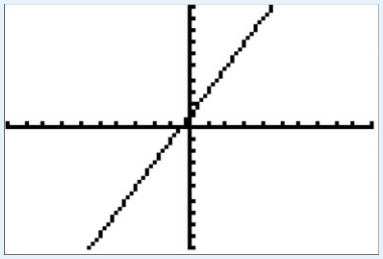
3.3 Finding the gradient (slope) of a line

The correct mathematical notation for gradient (slope) is $\frac{dy}{dx}$. You will find out more about this in the chapter on differential calculus. Here we just need to know this is the notation that will give us the gradient (slope) of the line.

Example 17

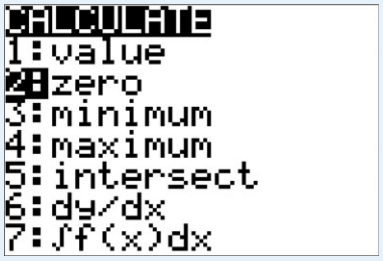
Find the gradient of $y = 2x + 1$.

First draw the graph of $y = 2x + 1$ (see Example 15).



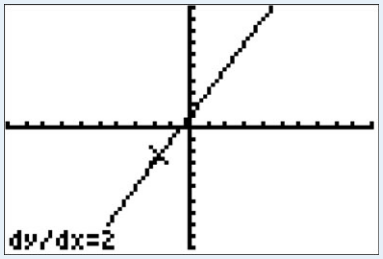
Press 2nd CALC | 6: dy/dx

Press ENTER



Select any point on the line using the ◀ and ▶ keys and press ENTER.

The gradient (slope) is 2.



3.4 Solving simultaneous equations graphically

To solve simultaneous equations graphically you draw the straight lines and then find their point of intersection. The coordinates of the point of intersection give you the solutions x and y.

Note: The calculator will only draw the graphs of functions that are expressed explicitly. By that we mean functions that begin with '$y =$' and have a function that involves only x to the right of the equals sign. If the equations are written in a different form, you will need to rearrange them before using your calculator to solve them.

Solving simultaneous equations using a non-graphical method is covered in section 1.1.

Example 18

Solve the simultaneous equations $2x + y = 10$ and $x - y = 2$ graphically with your GDC.

First rearrange both equations in the form $y =$

$2x + y = 10$ $\qquad$ $x - y = 2$

$y = 10 - 2x$ $\qquad$ $-y = 2 - x$

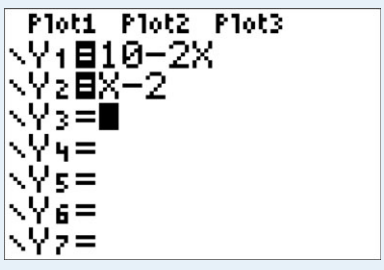
$y = x - 2$

To draw graphs $y = 10 - 2x$ and $y = x - 2$:

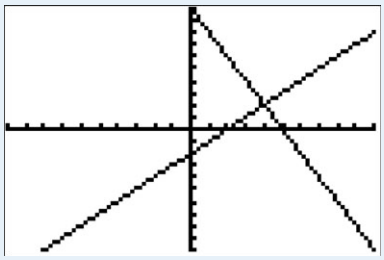
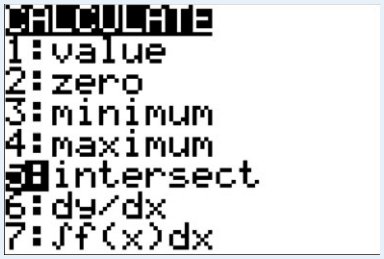
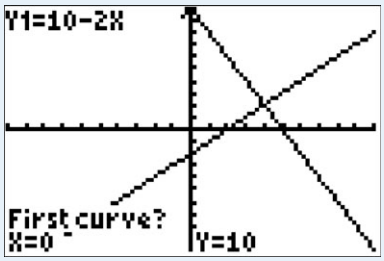
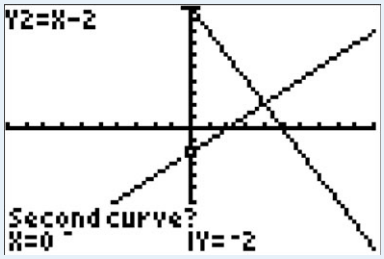
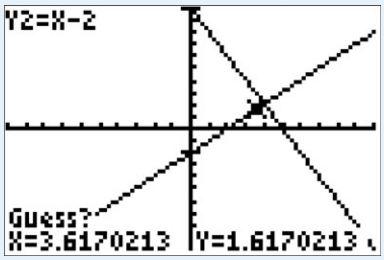
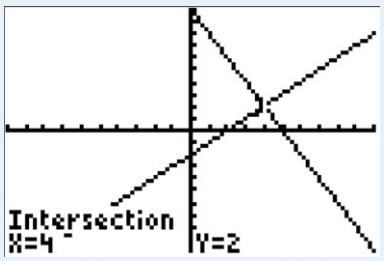
Press Y= to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

Type $10 - 2x$ and press ENTER and $x - 2$ and press ENTER.

Press ZOOM | 6:Z Standard to use the default axes which are $-10 \le x \le 10$ and $-10 \le y \le 10$.



▶ Continued on next page

The calculator displays both straight line graphs Y1 = 10 – 2x and Y2 = x – 2	
Press 2nd CALC | 5:intersect Press ENTER	
Press ENTER to select the first curve.	
Press ENTER to select the second curve.	
Select a point close to the intersection using the ◀ and ▶ keys and press ENTER.	
The calculator displays the intersection of the two straight lines at the point (4, 2). The solutions are $x = 4$, $y = 2$.	

4 Graphing quadratic and exponential functions

Quadratic functions

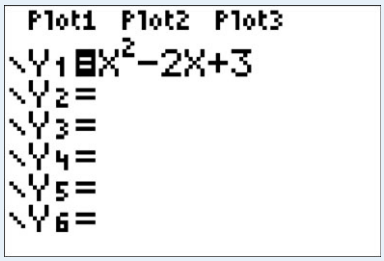
4.1 Drawing a quadratic graph

Example 19

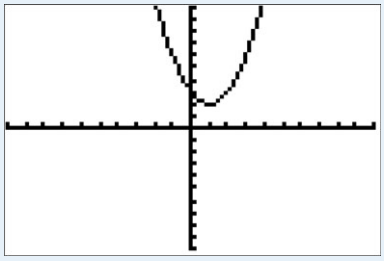
Draw the graph of $y = x^2 - 2x + 3$ and display it using suitable axes.

Press Y= to display the Y= editor. The default graph type is Function, so the form Y= is displayed.
Type $x^2 - 2x + 3$ and press ENTER.

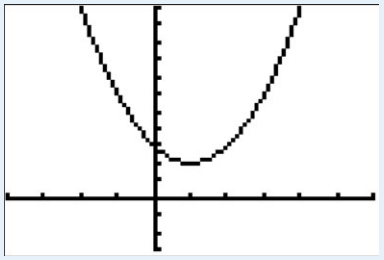
Press ZOOM | 6:Z Standard to use the default axes which are $-10 \le x \le 10$ and $-10 \le y \le 10$.



The calculator displays the curve with the default axes.



Adjust the window to make the quadratic curve fit the screen better.

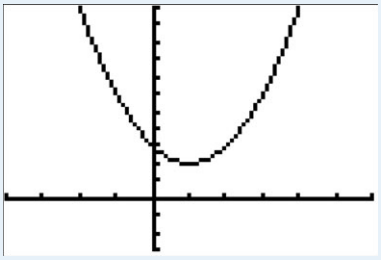


4.2 Finding a local minimum or maximum point

Example 20

Find the minimum point on the graph of $y = x^2 - 2x + 3$.

Draw the graph of $y = x^2 - 2x + 3$ (See Example 19).



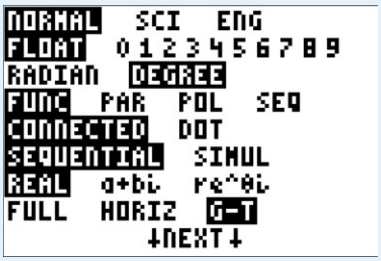
▶ Continued on next page

Method 1 - using a table

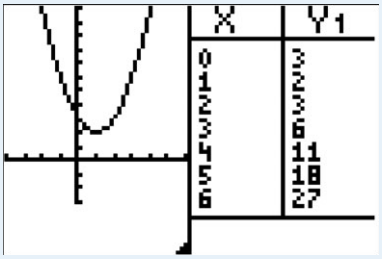
You can look at the graph and a table of the values on the graph by using a split screen.

Press MODE and select G-T

Press GRAPH



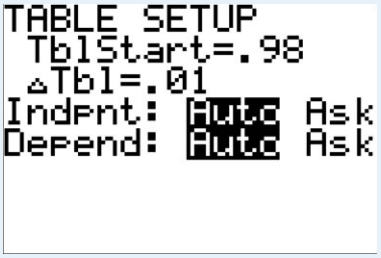
The minimum value shown in the table is 2 when $x = 1$.



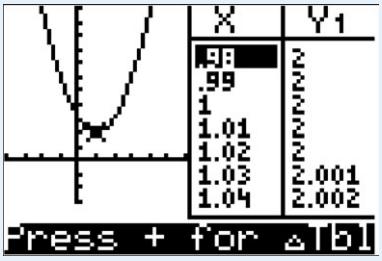
Look more closely at the values of the function around $x = 1$.

Change the settings in the table: Press 2nd TBLSET

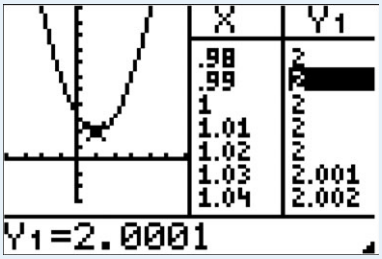
Set TblStart to 0.98
△Tbl to 0.01
Press 2nd TABLE to return to the graph and table screen.



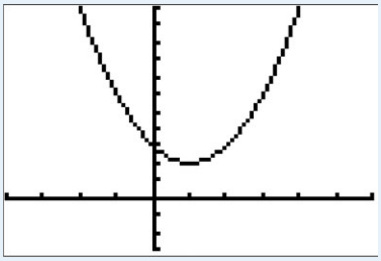
Press ▸ to move to the column containing y-values. This shows greater precision in the box below the table.



The table shows that the function has larger values at points around (1, 2). We can conclude that this is a local minimum on the curve.



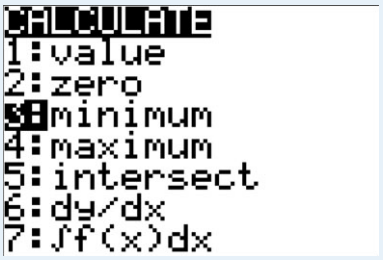
Method 2 - Using the minimum function



▶ Continued on next page

Press 2nd CALC | 3:minimum

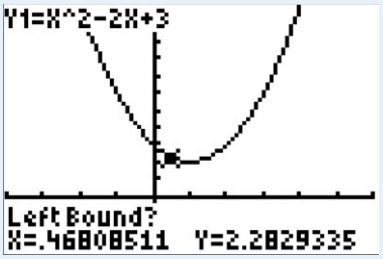
Press ENTER



To find the minimum point you need to give the left and right bounds of a region that includes it.

The calculator shows a point and asks you to set the left bound. Move the point using the ◀ and ▶ keys to choose a position to the left of the minimum.

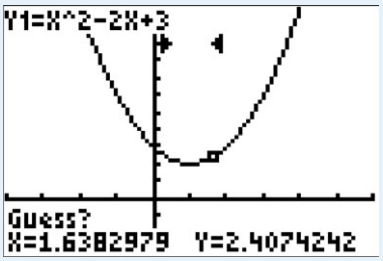
Press ENTER



The calculator shows another point and asks you to set the right bound.

Move the point using the ◀ and ▶ keys so that the region between the left and right bounds contains the minimum.

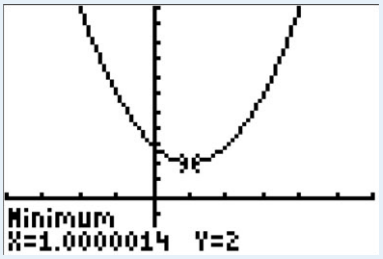
When the region contains the minimum press ENTER.



Press ENTER again to supply a guess for the value of the minimum.

The calculator displays the minimum point on the curve at $(1, 2)$.

In this example the value of x is not exactly 1. This is due to the way the calculator finds the point. You should ignore small errors like this when you write down the coordinates of the point.



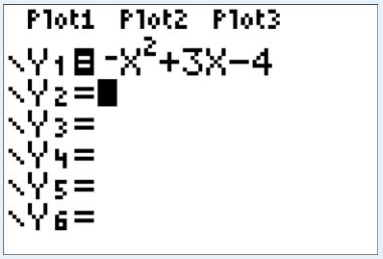
Example 21

Find the maximum point on the graph of $y = -x^2 + 3x - 4$.

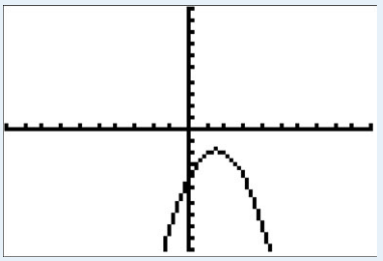
Press Y= to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

Type $-x^2 + 3x - 4$ and press ENTER.

Press ZOOM | 6:Z Standard to use the default axes which are $-10 \le x \le 10$ and $-10 \le y \le 10$.

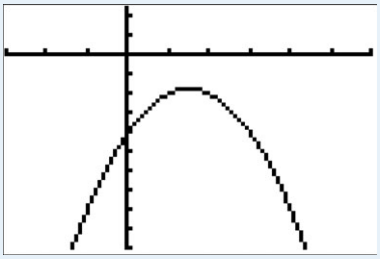


The calculator displays the curve with the default axes.



▶ Continued on next page

Adjust the window to make the quadratic curve fit the screen better.

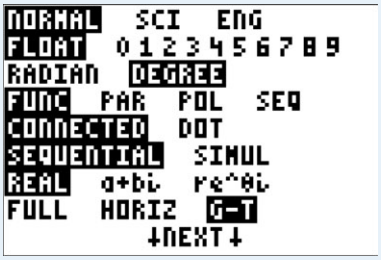


Method 1 - using a table

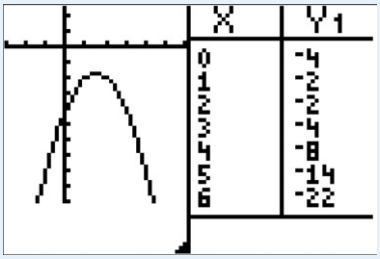
You can look at the graph and a table of the values on the graph by using a split screen.

Press MODE and select G-T

Press GRAPH



The maximum value shown in the table is –2 when $x = 1$ and $x = 2$.



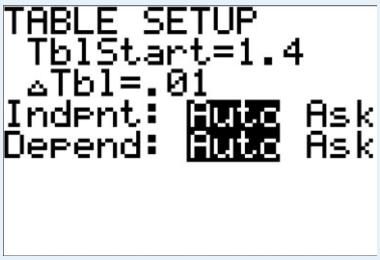
Look more closely at the values of the function between $x = 1$ and $x = 2$.

Change the settings in the table: Press 2nd TBLSET

Set TblStart to1.4

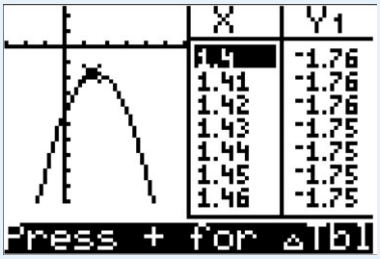
△Tbl to 0.01

Press 2nd TABLE to return to the graph and table screen.

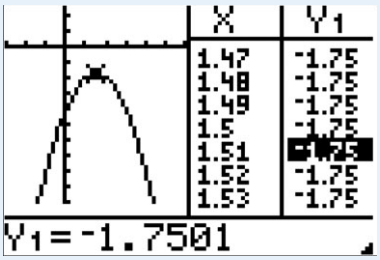


Press ▸ to move to the column containing y-values. This shows greater precision in the box below the table.

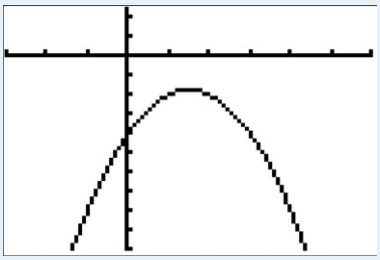
Press ▾ to scroll down until you find the maximum value of y.



The table shows that the function has smaller values at points around (1.5, –1.75). We can conclude that this is a local maximum on the curve.



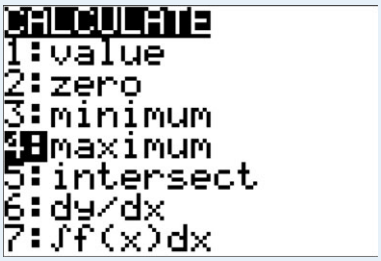
Method 2 - Using the maximum function



▶ Continued on next page

Press 2nd CALC | 4:maximum

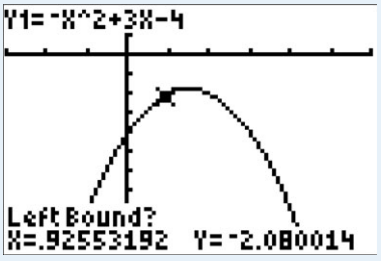
Press ENTER



To find the maximum point you need to give the left and right bounds of a region that includes it.

The calculator shows a point and asks you to set the left bound. Move the point using the ◀ and ▶ keys to choose a position to the left of the maximum.

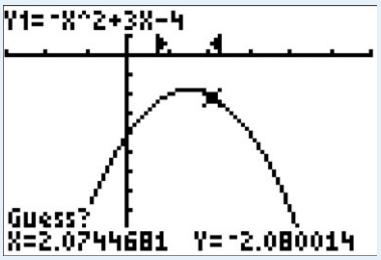
Press ENTER



The calculator shows another point and asks you to set the right bound.

Move the point using the ◀ and ▶ keys so that the region between the left and right bounds contains the minimum.

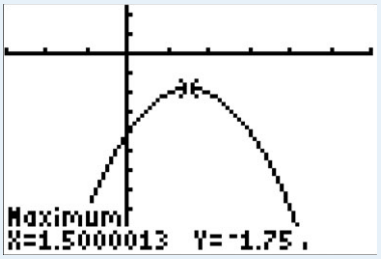
When the region contains the minimum press ENTER.



Press ENTER again to supply a guess for the value of the minimum.

The calculator displays the maximum point on the curve at (1.5, –1.75).

In this example the value of x is not exactly 1.5. This is due to the way the calculator finds the point. You should ignore small errors like this when you write down its coordinates.



Exponential functions

4.3 Drawing an exponential graph

Example 22

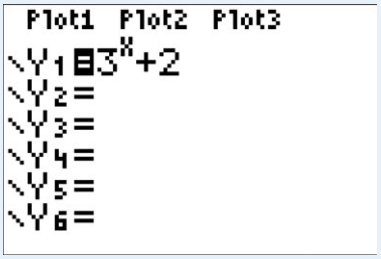
Draw the graph of $y = 3^x + 2$.

Press Y= to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

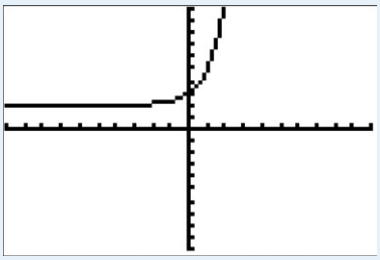
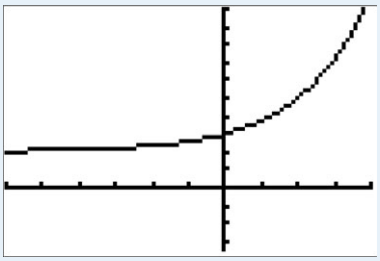
Type $3^x + 2$ and press ENTER.

(**Note**: Type 3 ^ X,T,Θ,n ▶ to enter 3^x. The ▶ returns you to the baseline from the exponent.)

Press ZOOM | 6:Z Standard to use the default axes which are $-10 \le x \le 10$ and $-10 \le y \le 10$.



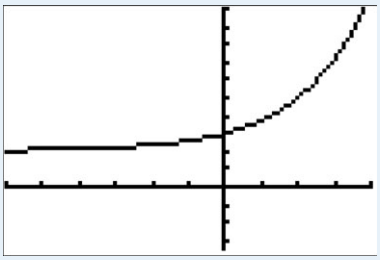
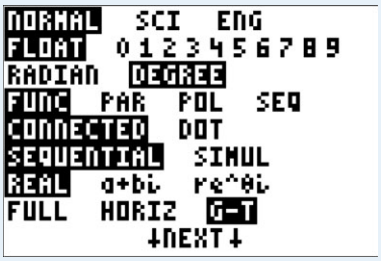
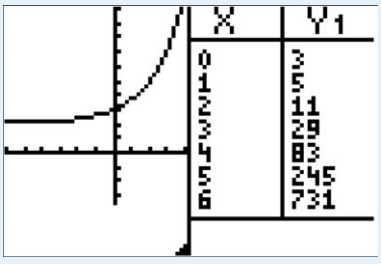
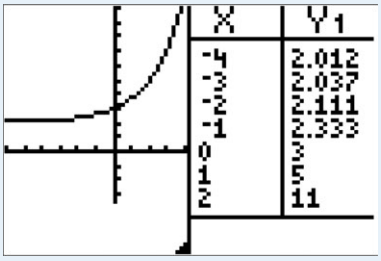
▶ Continued on next page

The calculator displays the curve with the default axes.	
Adjust the window to make the exponential curve fit the screen better.	

4.4 Finding a horizontal asymptote

Example 23

Find horizontal asymptote to graph of $y = 3^x + 2$.

Draw the graph of $y = 3^x + 2$ (See Example 22).	
You can look at the graph and a table of the values on the graph by using a split screen. Press MODE and select G-T Press GRAPH	
The values of the function are clearly decreasing as $x \to 0$.	
Press 2nd TABLE to switch to the table. Press ▲ to scroll up the table. The table shows that as the values of x get smaller, Y_1 approaches 2.	

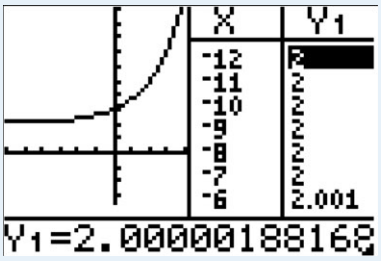
▶ Continued on next page

Eventually the value of Y_1 displayed in the table reaches 2.

Press ▸ to move to the column containing y-values. This shows greater precision in the box below the table. You can see, at the bottom of the screen, that the actual value of Y_1 is 2.00000188168...

We can say that $Y_1 \to 2$ as $x \to -\infty$.

The line $x = 2$ is a horizontal asymptote to the curve $y = 3^x + 2$.



More complicated functions

4.5 Solving a combined quadratic and exponential equation

Follow the same GDC procedure when solving simultaneous equations graphically and solving a combined quadratic and exponential equation. See Examples 18 and 24.

Example 24

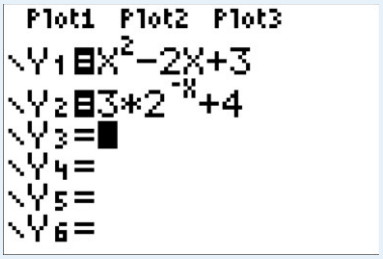
Solve the equation $x^2 - 2x + 3 = 3 \cdot 2^{-x} + 4$

To solve the equation, find the point of intersection between the quadratic function $y_1 = x^2 - 2x + 3$ and the exponential function $y_2 = 3 \times 2^{-x} + 3$.

Press Y= to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

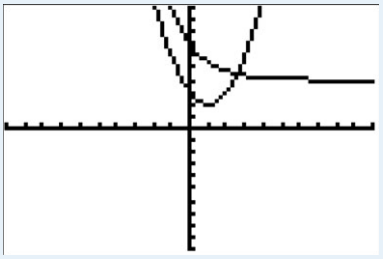
Type $x^2 - 2x + 3$ in Y_1 and press ENTER. Then type $3 \times 2^{-x} + 4$ in Y_2 and press ENTER.

(**Note**: Type 2 ∧ (−) X,T,Θ,n ▸ to enter 2^{-x}. The ▸ returns you to the baseline from the exponent.)

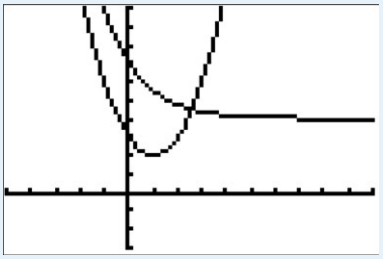


Press ZOOM | 6:Z Standard to use the default axes which are $-10 \le x \le 10$ and $-10 \le y \le 10$.

The calculator displays the curves with the default axes.

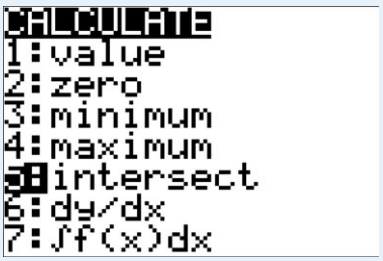


Adjust the window to make the quadratic curve fit the screen better.



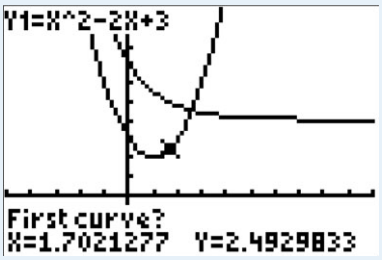
Press 2nd CALC | 5:intersect

Press ENTER

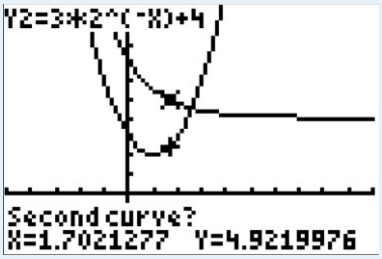


▶ Continued on next page

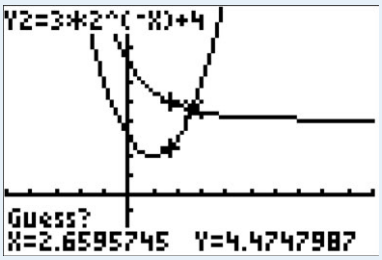
Press ENTER to select the first curve.



Press ENTER to select the second curve.

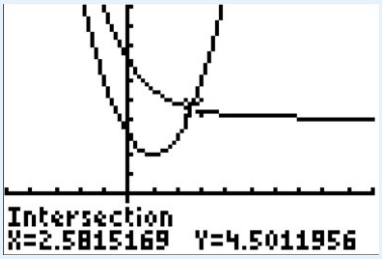


Select a point close to the intersection using the ◀ and ▶ keys and press ENTER.



The calculator displays the intersection of the two straight lines at the point (2.58, 4.50).

The solutions are $x = 2.58$ and $y = 4.50$.



5 Statistical applications

Calculating normal probabilities

5.1 Calculating normal probabilities from *X*-values

Example 25

A random variable X is normally distributed with a mean of 195 and a standard deviation of 20 or $X \sim N(195, 20^2)$. Calculate

a the probability that X is less than 190.

b the probability that X is greater than 194.

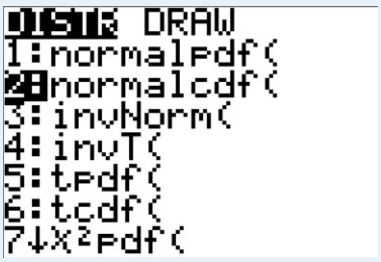
c the probability that X lies between 187 and 196.

Press 2nd DISTR | 2:normalcdf(

Press ENTER

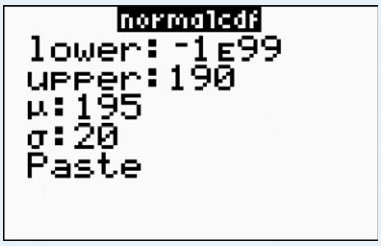
You should enter the values, Lower Bound, Upper Bound, μ and σ, in order.

The value 1E99 is the largest value that can be entered in the GDC and is used in the place of ∞. It stands for 1×10^{99} (–1E99 is the smallest value and is used in the place of –∞). To enter the E, you need to press 2nd EE.

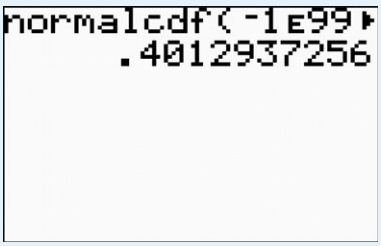


▶ Continued on next page

a $P(X < 190)$
Enter lower bound as −1E99, upper bound as 190, μ to 195 and σ to 20.
Select Paste and press ENTER

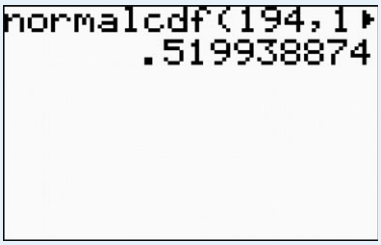


press ENTER again
P(X < 190) = 0.401 (to 3 sf)



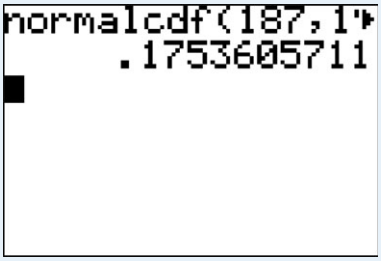
b $P(X > 194)$
Enter Lower Bound as 194, Upper Bound as 1E99, μ as 195 and σ as 20.
Select Paste and press ENTER

press ENTER again
P(X > 194) = 0.520 (to 3 sf)



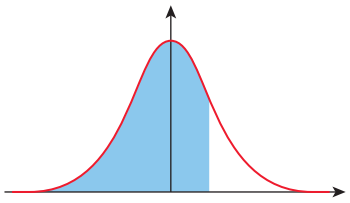
c $P(187 < X < 196)$
Enter Lower Bound as 187, Upper Bound as 196, μ as 195 and σ as 20.

press ENTER again
P(187 < X < 196) = 0.175 (to 3 sf)

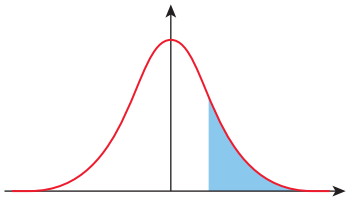


5.2 Calculating *X*-values from normal probabilities

In some problems you are given probabilities and have to calculate the associated values of X. To do this, use the invNorm function.



When using the Inverse Normal function, make sure you find the probability on the correct side of the normal curve. The areas are always the lower tail, that is they are always in the form $P(X < x)$ (see Example 26).



If you are given the upper tail $P(X > x)$, you must first subtract the probability from 1 before you can use invNorm (see Example 27).

Example 26

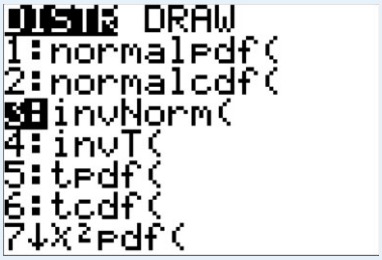
A random variable X is normally distributed with a mean of 75 and a standard deviation of 12 or $X \sim N(75, 12^2)$. If $P(X < x) = 0.4$, find the value of x.

You are given a lower-tail probability so you can find $P(X < x)$ directly.

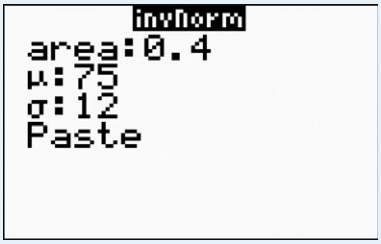
Press 2nd DISTR | 3:invNorm(

Press ENTER

You should enter the values: area (probability), μ and σ, in order.

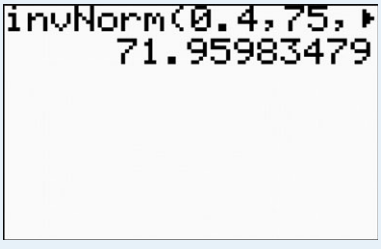


Enter area as 0.4, μ as 75 and σ as 12.



press ENTER again

So if $P(X < x) = 0.4$ then $x = 72.0$ (to 3 sf).



Example 27

A random variable X is normally distributed with a mean of 75 and a standard deviation of 12 or $X \sim N(75, 12^2)$. If $P(X > x) = 0.2$, find the value of x.

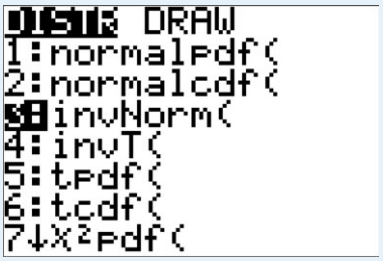
You are given an upper-tail probability so you must first find $P(X < x) = 1 - 0.2 = 0.8$. You can now use the invNorm function as before.

▶ Continued on next page

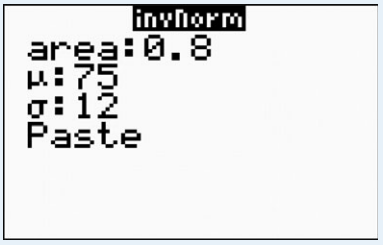
Press 2nd DISTR | 3:invNorm(

Press ENTER

You should enter the values: area (probability), μ and σ, in order.

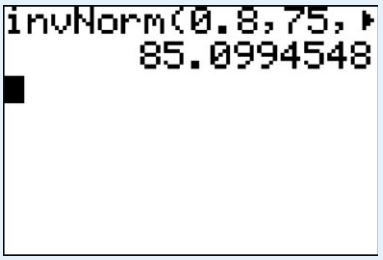


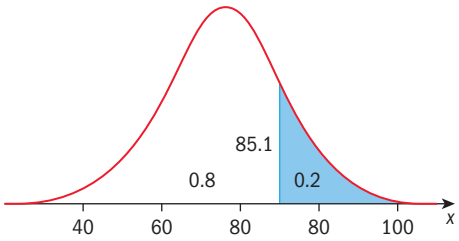
Enter area as 0.8, μ as 75 and σ as 12.



press ENTER again

So if $P(X > x) = 0.2$ then $x = 85.1$ (to 3sf).





▲ This sketch of a normal distribution curve shows this value and the probabilities from Example 27.

Scatter diagrams, linear regression and the correlation coefficient

5.3 Scatter diagrams

Example 28

Consider this data that is approximately connected by a linear function.

x	1.0	2.1	2.4	3.7	5.0
y	4.0	5.6	9.8	10.6	14.7

a Find the equation of the least squares regression line of y on x

b Find Pearson's product-moment correlation coefficient.

c Use the equation to predict the value of y when $x = 3.0$.

▶ Continued on next page

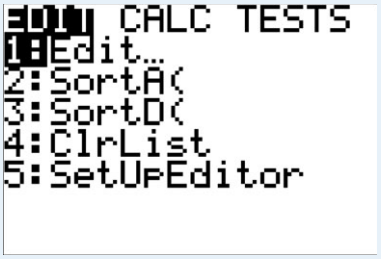
Press STAT | 1:Edit and press ENTER.

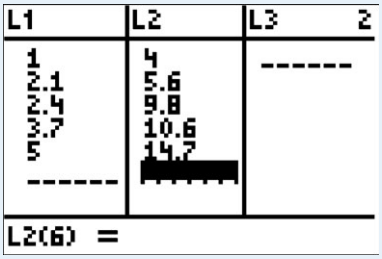
Type the values of x in the first column (L1), and the values of y in the second column (L2).

Press ENTER or ▼ after each number to move down to the next cell.

Press ▶ to move to the next column.

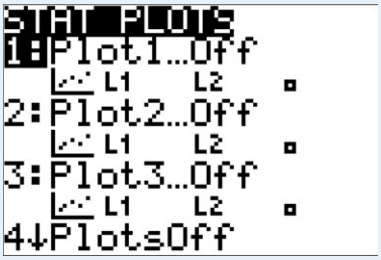
You can use columns from L1 to L6 to enter the lists.

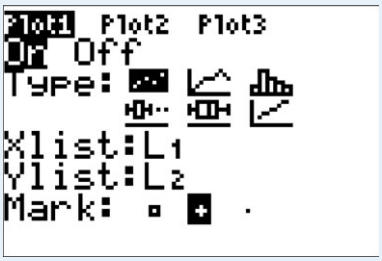




Press 2nd STAT PLOT and ENTER to select Plot1.

Select On, choose the scatter diagram option, XList as L1 and Ylist as L2. You can choose one of the three types of mark.

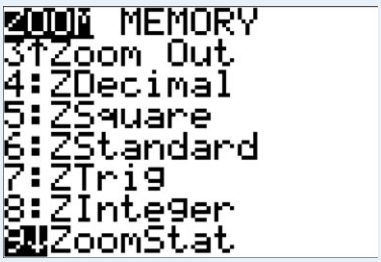


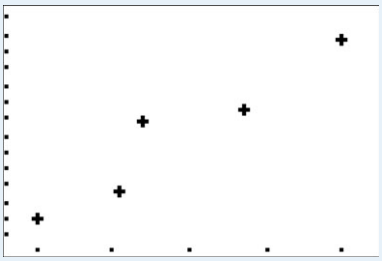


Press ZOOM | 9:Stat

The automatic scales do not usually give the best display of the scatter diagram. You will need to change the default values.

You may need to delete any function graphs. Y=





▶ Continued on next page

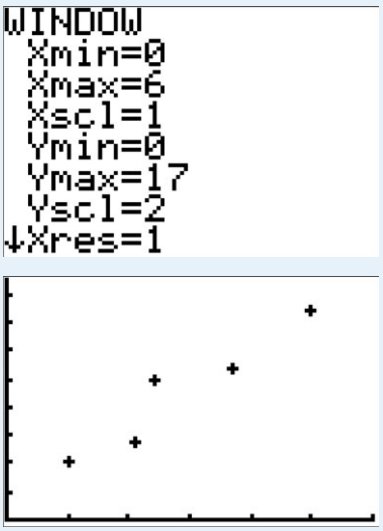
Press WINDOW and choose options as shown.

Xmin and Xmax should include the range of the x-data.

Ymin and Ymax should include the range of the y-data.

You now have a scatter graph of y against x.

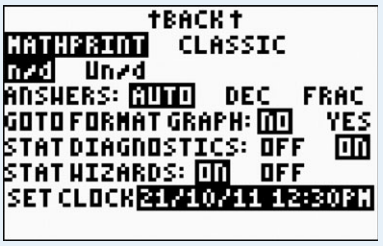
You need to include zero in the range if you want to show the axes on the graph.



In order to see the Pearson's product-moment correlation coefficient, you need to have diagnostics on your GDC switched on.

Press MODE and use ▼ to scroll down to the second screen. Set STAT DIAGNOSTICS to ON and press ENTER.

Then press 2nd QUIT to return to the home screen.



Press STAT | CALC |4:LinReg($ax + b$)

Enter Xlist as L1, Ylist as L2 and leave FreqList blank

In Store RegEQ press ALPHA F4 and press ENTER to select Y1

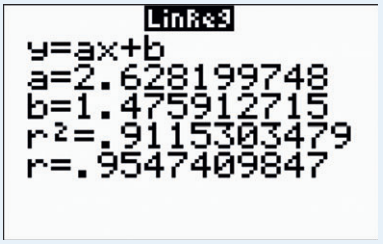
Select Calculate and press ENTER.

You will see the coefficients of the equation of the least squares regression line and the value r of the correlation coefficient.

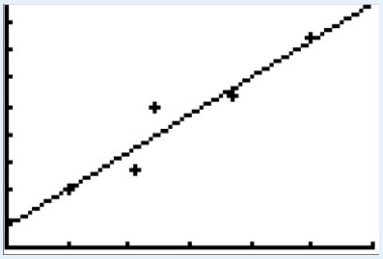
The equation is $y = 2.63x + 1.48$ (to 3sf).

The value of r is 0.955 (to 3sf).

The coefficient of determination, $r2$, is not used in Mathematical Studies.



Press GRAPH and you will see the least squares regression line and the data points that you plotted previously.



▶ Continued on next page

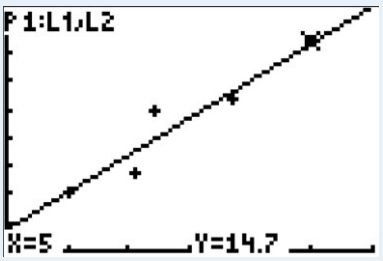
Press TRACE and use the ▶ ◀ keys to move the trace along the line.

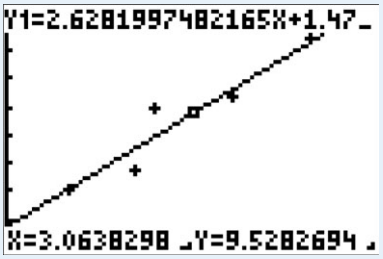
The cursor moves between the data points.

Press ▲ to move onto the line itself.

It is not possible to move the trace point to an exact value, so get as close to $x = 3$ as you can.

From the graph, you have found that y is approximately 9.5 when $x = 3.0$.

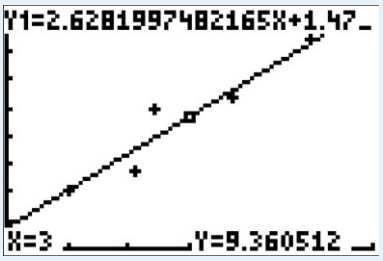




Press 3 ENTER.

The cursor now moves to exactly 3.0.

When $x = 3.0$, an estimate of the value of y is 9.36, from the graph.



The χ^2 test for independence

5.4 Using contingency tables

Data from a contingency table is entered into a matrix. The calculator will calculate the expected frequencies, χ^2 value, the number of degrees of freedom and the p-value.

Example 29

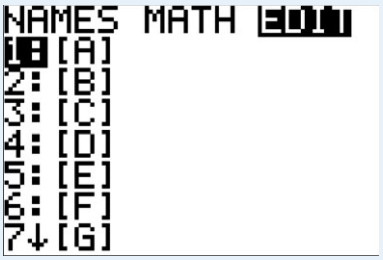
A survey of the favorite colour for a mobile phone produced the following data.

	Black	Red	Blue	Silver
Male	48	35	33	54
Female	35	66	42	27

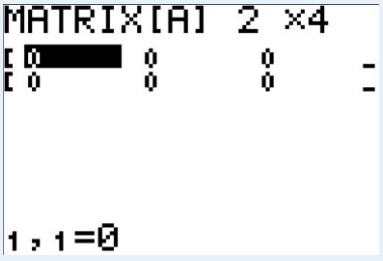
Test to find out if the choice of colour is independent of gender at the 5% level.

Press 2nd MATRIX and use the ▶ to select the EDIT menu.

Press ENTER.



Press 2 ENTER 4 ENTER to choose 2 rows and 4 columns.

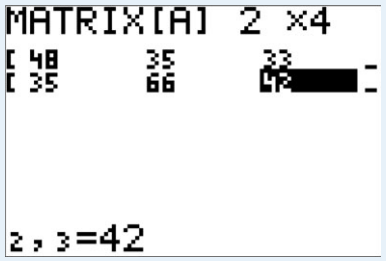


▶ Continued on next page

Enter the data from the table above in the matrix.

Press ENTER after each.
The matrix [A] now contains the "observed frequencies" for the χ^2 calculations.

The last column of the matrix is not displayed as there is not enough room on the screen.



Press 2nd QUIT to return to the home screen.

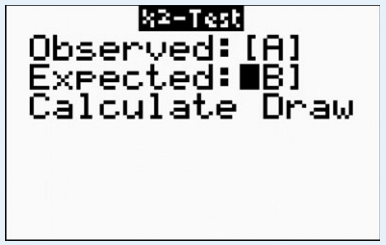
Press STAT and use the ▶ to select the TESTS menu.

Select C:χ^2-Test...

Press ENTER.

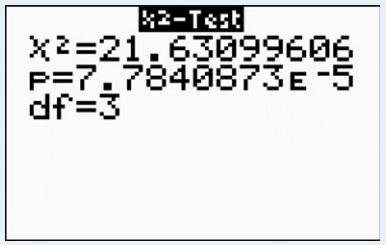
If the data is entered in matrix [A], you will not need to change anything in this screen.

Use the ▼ key to scroll down to Calculate and press ENTER.



This shows that the χ^2 value was 21.631, the p-value was 0.000078 and the number of degrees of freedom is 3.

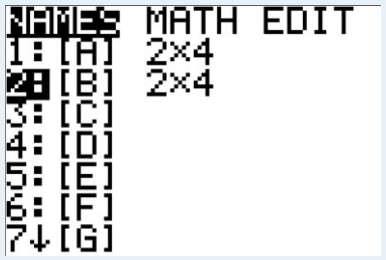
Since 0.000078 < 0.05, we would not accept the null hypothesis that the favorite colour of phone was independent of gender.



When you have finished, you should always check the table of expected frequencies, to ensure that they are all greater than 5. These values are in the matrix [B].

Press 2nd MATRIX and use the ▼ to select the 2:[B]

Press ENTER



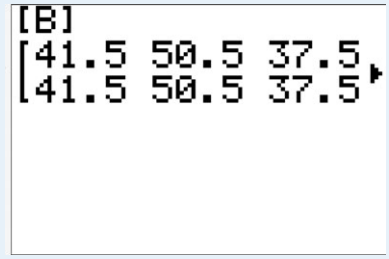
This pastes [B] on the home screen.

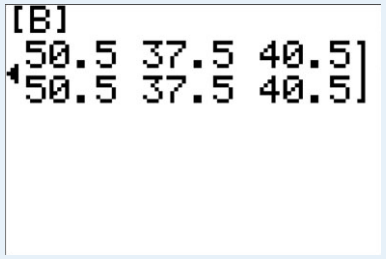
Press ENTER again.

Here, none of the entries is less than 5.

Note: If there are too many expected values between 1 and 5, you can combine rows or columns in the table.

Use ▶ to scroll right and see the fourth column of the matrix. You must do this before you press any other keys on the GDC.





6 Differential calculus

Finding gradients, tangents and maximum and minimum points

6.1 Finding the gradient at a point

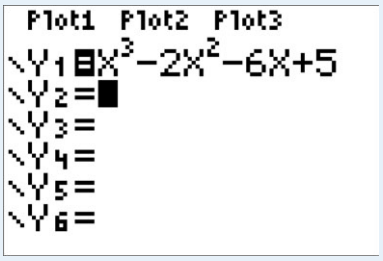
Example 30

Find the gradient of the cubic function $y = x^3 - 2x^2 - 6x + 5$ at the point where $x = 1.5$.

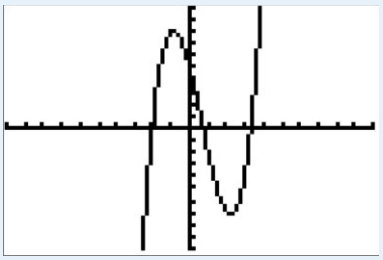
Press [Y=] to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

Type $y = x^3 - 2x^2 - 6x + 5$ and press [ENTER].

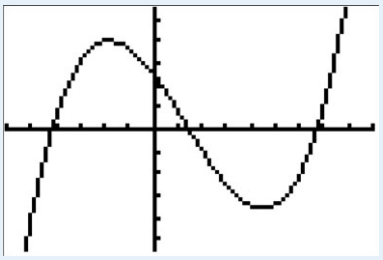
(Note: Type [X,T,Θ,n] [^] [3] [▸] to enter x^3. The [▸] returns you to the baseline from the exponent.)



Press [ZOOM] | 6:ZStandard to use the default axes which are $-10 \le x \le 10$ and $-10 \le y \le 10$.



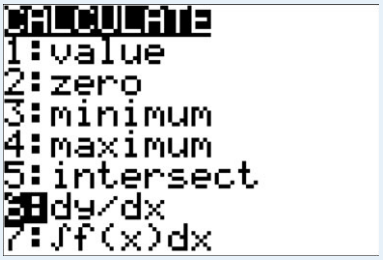
Adjust the window to make the cubic curve fit the screen better.



Press [2nd] [CALC] | 6: dy/dx

Press [ENTER]

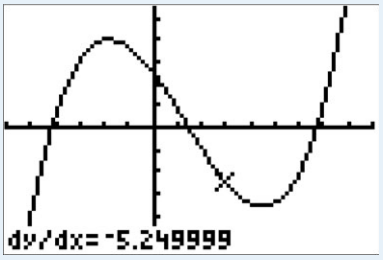
Press [1] [.] [5] [ENTER]



The calculator displays the gradient of the curve at the point where $x = 1.5$.

The gradient is –5.25.

In this example the value of xdy/dx is not exactly –5.25. This is due to the way the calculator finds the point gradient. You should ignore small errors like this when you write down the coordinates of a gradient at a point.

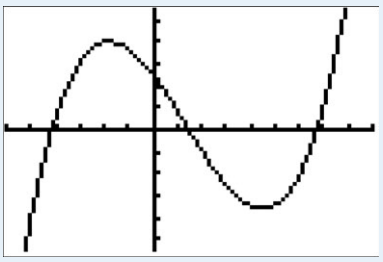


6.2 Drawing a tangent to a curve

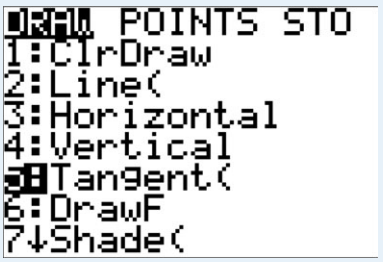
Example 31

Draw a tangent to the curve $y = x^3 - 2x^2 - 6x + 5$ where $x = -0.5$.

First draw the graph of $y = x^3 - 2x^2 - 6x + 5$ (see Example 30).

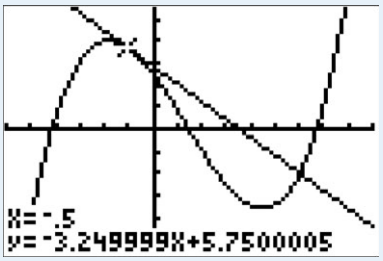


Press 2nd DRAW
Choose 5:Tangent
Press ENTER



Press (-) 0 . 5 ENTER
The equation of the tangent is
$y = -3.25x + 5.75$.

In this example the values −3.25 and 5.75 are not shown as being exact. This is due to the way the calculator finds the values. You should ignore small errors like this when you write down the equation of a tangent.

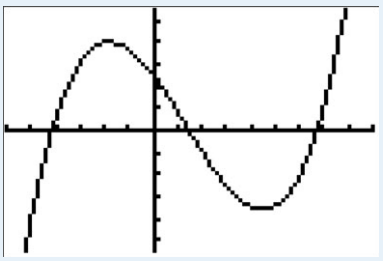


6.3 Finding maximum and minimum points

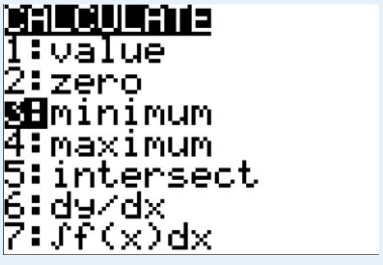
Example 32

Find the local maximum and local minimum points on the cubic curve.

First draw the graph of $y = x^3 - 2x^2 - 6x + 5$ (see Example 30).



Press 2nd CALC | 3:minimum
Press ENTER

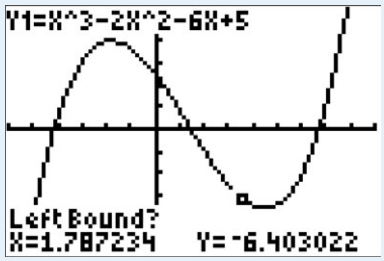


▶ Continued on next page

To find the minimum point you need to give the left and right bounds of a region that includes it.

The calculator shows a point and asks you to set the left bound. Move the point using the ◀ and ▶ keys to choose a position to the left of the minimum.

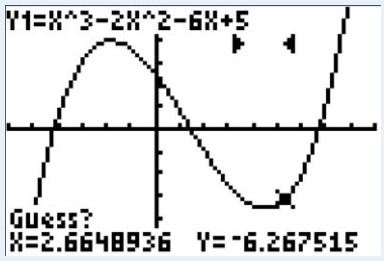
Press ENTER.



The calculator shows another point and asks you to set the right bound.

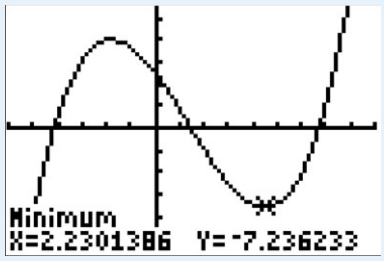
Move the point using the ◀ and ▶ keys so that the region between the left and right bounds contains the minimum.

When the region contains the minimum press ENTER.

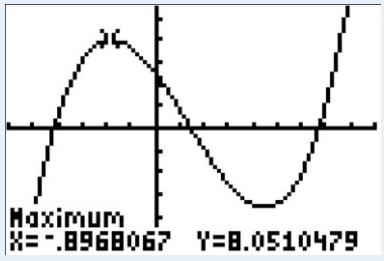


Press ENTER again to supply a guess for the value of the minimum.

The calculator displays the local minimum at the point (2.23, −7.24).



Press 2nd CALC | 3:maximum ENTER to find the local maximum point on the curve in exactly the same way.
The maximum point is (−0.897, 8.05).



7 Number and algebra 2

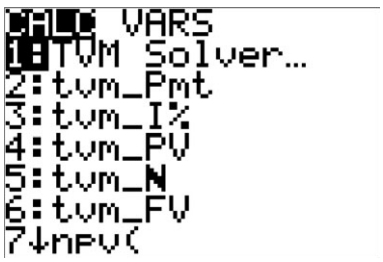
The finance solver

The Finance Solver will solve problems about simple loans, mortgages, and investments.

In general in financial problems, a negative monetary amount indicates an amount you give to the bank and a positive monetary amount indicates money you receive from the bank. This can be a little confusing.

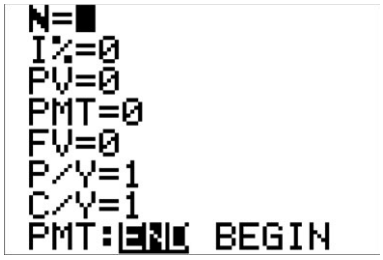
Press APPS 1:Finance...

Choose 1:TVM Solver...
Press ENTER



You will see this dialog box, where:

N: is the total number of payments.
I(%): is the annual interest rate as a percent.
PV: is the present value, which is negative for investments.
PMT : is the payment or regular deposit, which is negative for investments.
FV: is the future value.
P/Y: is the payments per year.
C/Y: is the interest calculations period per year.
PMT: is payments made at the end or beginning of each period.



7.1 Finding the total value of an investment

Example 33

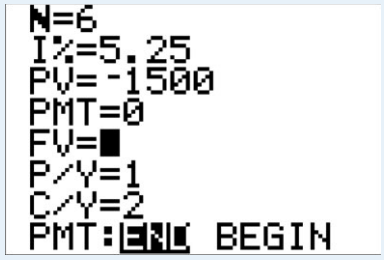
$1500 is invested at a rate of 5.25% per annum. The interest is compounded two times per year. How much will the investment be worth at the end of 6 years?

Press APPS 1:Finance... | 1:TVM Solver... ENTER

Enter values into the table.

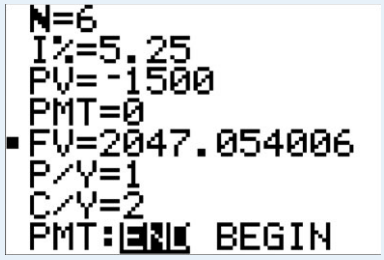
Press ENTER after entering each value to move through the table.

N: 6
I(%): 5.25
PV: −1500
PMY: 0
FV: To be calculated
P/Y: 1
C/Y: 2
PMT: END



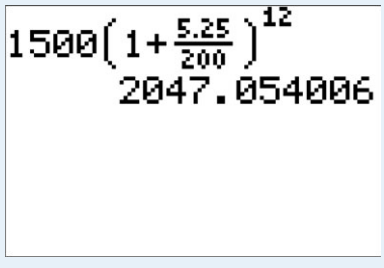
Select FV and press ALPHA SOLVE

The final amount is $2047.05.



Note: This result can also be found by using the calculation:

$$1500\left(1+\frac{5.25}{100}\right)^{12}$$



7.2 Calculating payments for a loan

Example 34

Calculate the monthly payment to repay a 4-year loan of $12,000 that is compounded monthly at an annual interest rate of 4.25%. Payments are made at the end of each month.

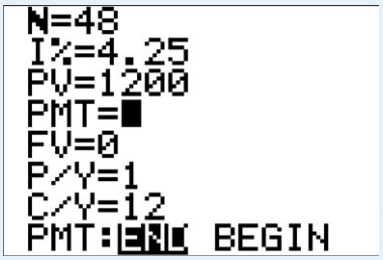
▶ Continued on next page

Press APPS 1:Finance... | 1:TVM Solver... ENTER
Enter values into the table.

Press ENTER after entering each value to move through the table.

N: 48
I(%): 5.25
PV: 12000
PMT: To be calculated
FV: 0
P/Y: 12
C/Y: 12
PMT: END

The repayments are made each month so the total number of payments, N, is 4 years × 12 = 48 months.



Select PMT and press ALPHA SOLVE

The monthly payments will be $272.29.
Note: The answer, PMT, is negative because it is a payment made to a bank.