

17

Using a graphic display calculator

CHAPTER OBJECTIVES:

This chapter shows you how to use your graphic display calculator (GDC) to solve the different types of problems that you will meet in your course. You should not work through the whole of the chapter – it is simply here for reference purposes. When you are working on problems in the mathematical chapters, you can refer to this chapter for extra help with your GDC if you need it.

Instructions for the TI-84 Plus calculator

Use this list to help you to find the topic you need

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Before you start

You should be familiar with:

- Important keys on the keyboard: **ON** **2nd** **DEL** **CLEAR** **Y=** **X,T,θ,n** **ENTER** **GRAPH**
- The home screen
- Changing window settings in the graph screen
- Using zoom tools in the graph screen
- Using trace in the graph screen

For a reminder of how to perform the basic operations have a look at your GDC manual.

1 Functions

1.1 Graphing linear functions

Example 1

Draw the graph of the function $y = 2x + 1$.

Press **Y=** to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

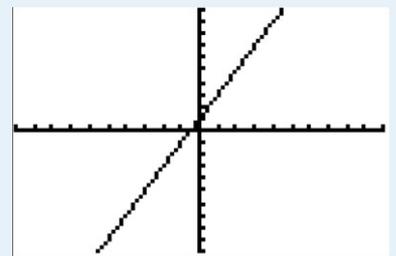
Type $2x + 1$ and press **ENTER**.

Press **ZOOM** | 6:ZStandard to use the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

```

Plot1 Plot2 Plot3
\Y1=2X+1
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
  
```

The graph of $y = 2x + 1$ is now displayed on the screen.



Finding information about the graph

The GDC can give you a lot of information about the graph of a function, such as the coordinates of points of interest and the gradient (slope).

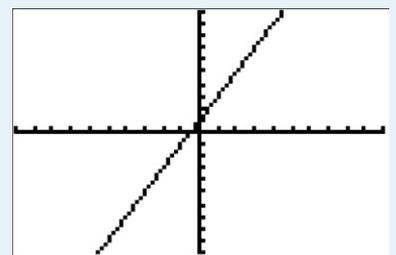
1.2 Finding a zero

The x -intercept is known as a *zero* of the function.

Example 2

Find the zero of $y = 2x + 1$.

Draw the graph of $y = 2x + 1$ as in Example 15.



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Press **2nd** **CALC** | 2:Zero.

Press **ENTER**.

```

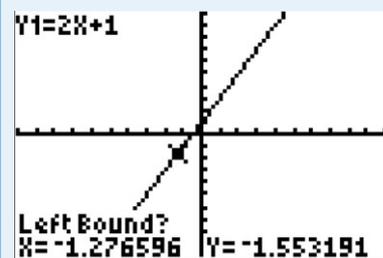
CALCULATE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx
  
```

To find the zero you need to give the left and right bounds of a region that includes the zero.

The calculator shows a point and asks you to set the left bound.

Move the point using the **◀** and **▶** keys to choose a position to the left of the zero.

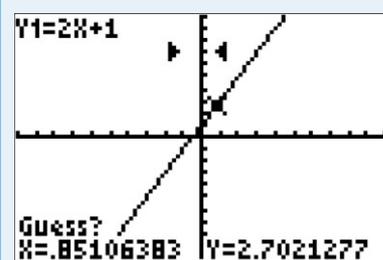
Press **ENTER**.



The calculator shows another point and asks you to set the right bound.

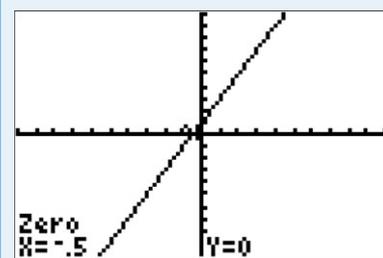
Move the point using the **◀** and **▶** keys so that the region between the left and right bounds contains the zero.

When the region contains the zero press **ENTER**.



Press **ENTER** again to supply a guess for the value of the zero.

The calculator displays the zero of the function $y = 2x + 1$ at the point $(-0.5, 0)$.



1.3 Finding the gradient (slope) of a line

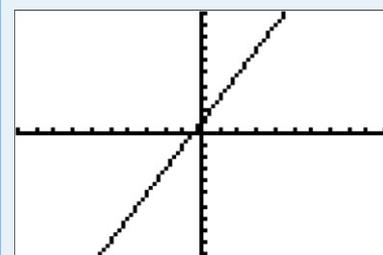
The correct mathematical notation for gradient (slope) is $\frac{dy}{dx}$. You will find

out more about this in the chapter on differential calculus. Here we just need to know this is the notation that will give us the gradient (slope) of the line.

Example 3

Find the gradient of $y = 2x + 1$.

First draw the graph of $y = 2x + 1$ (see Example 15).



▶ Continued on next page

Press **2nd** **CALC** | 6: dy/dx .

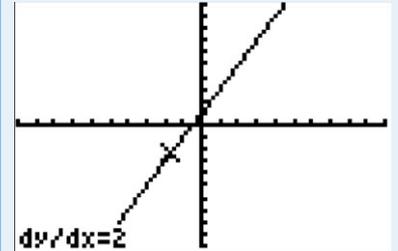
Press **ENTER**.

```

CALCULATE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx
  
```

Select any point on the line using the **◀** and **▶** keys and press **ENTER**.

The gradient (slope) is 2.



1.4 Solving simultaneous equations graphically

To solve simultaneous equations graphically you draw the straight lines and then find their point of intersection. The coordinates of the point of intersection give you the solutions x and y .

Note: The calculator will only draw the graphs of functions that are expressed explicitly. By that we mean functions that begin with 'y =' and have a function that involves only x to the right of the equals sign. If the equations are written in a different form, you will need to rearrange them before using your calculator to solve them.

Solving simultaneous equations using a non-graphical method is covered in section 1.5.

Example 4

Solve the simultaneous equations $2x + y = 10$ and $x - y = 2$ graphically with your GDC.

First rearrange both equations in the form $y =$

$$\begin{array}{ll} 2x + y = 10 & x - y = 2 \\ y = 10 - 2x & -y = 2 - x \\ & y = x - 2 \end{array}$$

To draw graphs $y = 10 - 2x$ and $y = x - 2$.

Press **Y=** to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

Type $10 - 2x$ and press **ENTER** and $x - 2$ and press **ENTER**.

Press **ZOOM** | 6:Z Standard to use the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

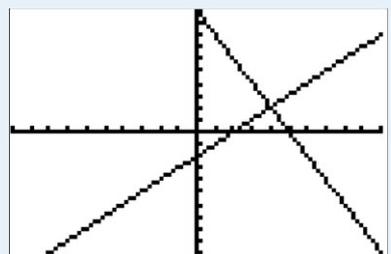
```

Plot1 Plot2 Plot3
\Y1=10-2X
\Y2=X-2
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
  
```

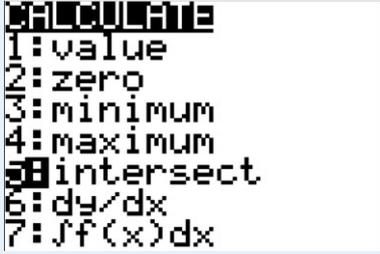
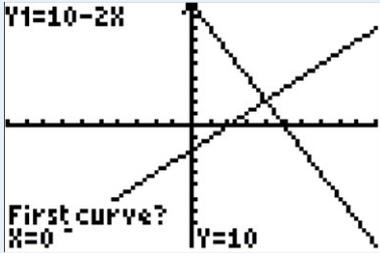
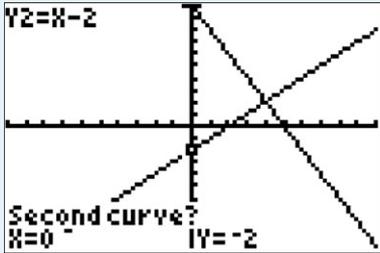
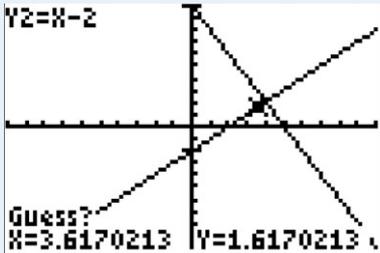
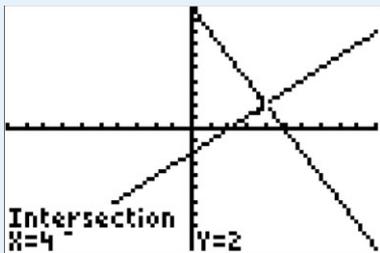
The calculator displays both straight line graphs

$$Y1 = 10 - 2x \text{ and}$$

$$Y2 = x - 2$$



▶ Continued on next page

<p>Press 2nd CALC 5:intersect.</p> <p>Press ENTER.</p>	
<p>Press ENTER to select the first curve.</p>	
<p>Press ENTER to select the second curve.</p>	
<p>Select a point close to the intersection using the ◀ and ▶ keys and press ENTER.</p>	
<p>The calculator displays the intersection of the two straight lines at the point (4, 2).</p> <p>The solutions are $x = 4$, $y = 2$.</p>	

Simultaneous and quadratic equations

1.5 Solving simultaneous linear equations

When solving simultaneous equations in an examination, you do not need to show any method of solution. You should simply write out the equations in the correct form and then give the solutions. The calculator will do all the working for you.

You will need to have the App PlySmlt2 installed on your GDC. This App is permitted by IBO in your examination.

Example 5

Solve the equations:

$$2x + y = 10$$

$$x - y = 2$$

Press **APPS**. You will see the dialog box as shown on the right. Choose the App PlySmlt2 and press **ENTER**.

```
APPLICATIONS
1: Finance...
2: CtlgHelp
3: PlySmlt2
```

From the main menu, choose 2: SIMULT EQN SOLVER and press **ENTER**.

```
MAIN MENU
1: POLY ROOT FINDER
2: SIMULT EQN SOLVER
3: ABOUT
4: POLY HELP
5: SIMULT HELP
6: QUIT POLYSMLT
```

The defaults are to solve two equations in two unknowns.

Note: This is how you will use the linear equation solver in your examinations. In your project, you might want to solve a more complicated system with more equations and more variables.

```
SIMULT EQN SOLVER MODE
EQUATIONS 3 4 5 6 7 8 9 10
UNKNOWN 2 3 4 5 6 7 8 9 10
DEC F1 F2 F3 F4 F5 F6 F7 F8 F9
NORMAL SCI ENG
FLOAT 0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
(MAIN) (HELP) (NEXT)
```

Press **F5** and you will see the template on the right.

Type the coefficients from two equations into the template, pressing **ENTER** after each number.

The equations must be in the correct order.

```
SYSTEM MATRIX (2x3)
[0 0 | 0 1]
[0 0 | 0 1]

(1,1)=0
(MAIN) (MODE) (CLR) (LOAD) (SOLVE)
```

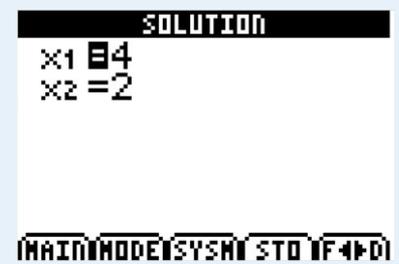
Press **F5** and the calculator will solve the equations, giving the solutions in the as x_1 and x_2 .

```
SYSTEM MATRIX (2x3)
[2 1 | 10 ]
[1 -1 | 2 ]

(2,3)=2
(MAIN) (MODE) (CLR) (LOAD) (SOLVE)
```

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The solutions are $x = 4, y = 2$.



Quadratic functions

1.6 Drawing a quadratic graph

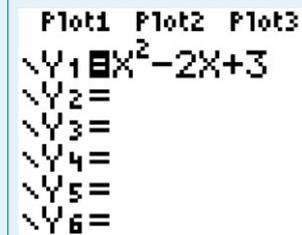
Example 6

Draw the graph of $y = x^2 - 2x + 3$ and display it using suitable axes.

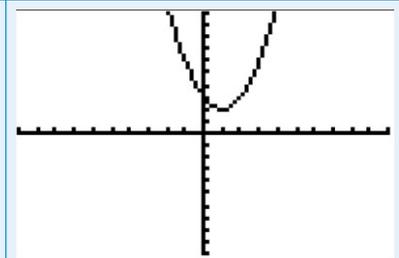
Press **Y=** to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

Type $x^2 - 2x + 3$ and press **ENTER**.

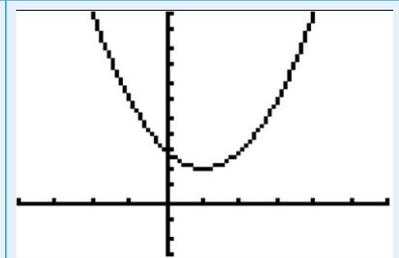
Press **ZOOM** | 6:Z Standard to use the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.



The calculator displays the curve with the default axes.



Adjust the window to make the quadratic curve fit the screen better.



1.7 Solving quadratic equations

When solving quadratic equations in an examination, you do not need to show any method of solution. You should simply write out the equations in the correct form and then give the solutions. The GDC will do all the working for you.

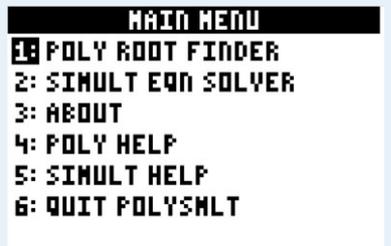
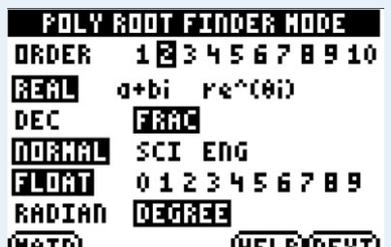
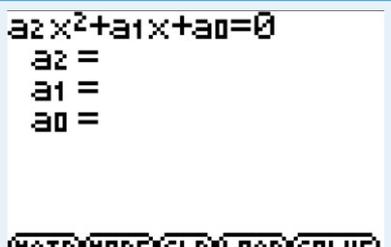
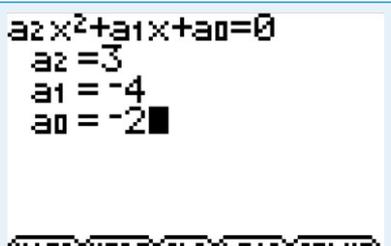
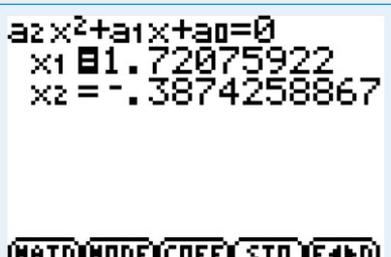
Example 7

Solve $3x^2 - 4x - 2 = 0$

Press **APPS**. You will see the dialog box as shown on the right. Choose the App PlySmlt2 and press **ENTER**.



▶ Continued on next page

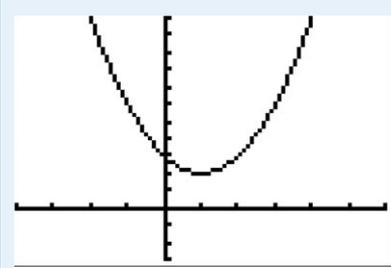
<p>From the main menu, choose 1: POLY ROOT FINDER and press ENTER.</p>	
<p>The defaults are to solve an equation of order 2 (a quadratic equation) with real roots. You do not need to change anything.</p>	
<p>Another dialog box opens for you to enter the equation. The general form of the quadratic equation is $a_2x^2 + a_1x + a_0 = 0$, so we enter the coefficients in a_2, a_1 and a_0.</p>	
<p>Here $a_2 = 3$, $a_1 = -4$ and $a_0 = -2$. Be sure to use the (-) key to enter the negative values. Press ENTER after each value. Press F5 and the calculator will find the roots of the equation.</p>	
<p>The solutions are $x = -0.387$ or $x = 1.72$ (3sf).</p>	

1.8 Finding a local minimum or maximum point

Example 8

Find the minimum point on the graph of $y = x^2 - 2x + 3$.

Draw the graph of $y = x^2 - 2x + 3$ (See Example 19).



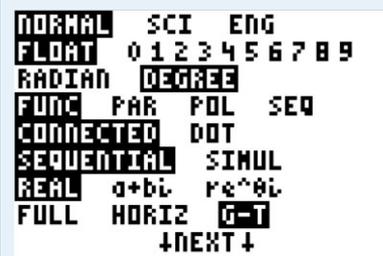
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Method 1 - using a table

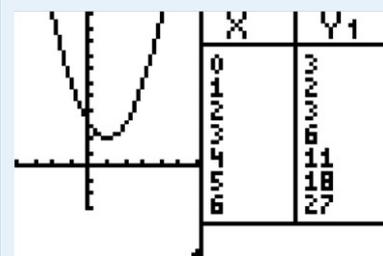
You can look at the graph and a table of the values on the graph by using a split screen.

Press **MODE** and select G-T.

Press **GRAPH**.



The minimum value shown in the table is 2 when $x = 1$.



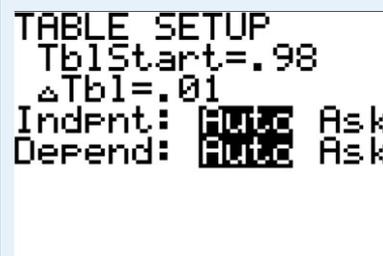
Look more closely at the values of the function around $x = 1$.

Change the settings in the table: Press **2nd** **TBLSET**.

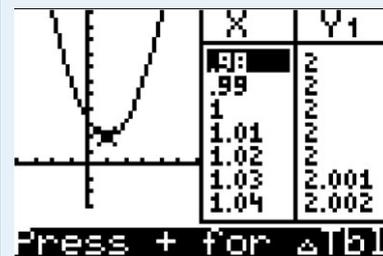
Set TblStart to 0.98

Δ Tbl to 0.01

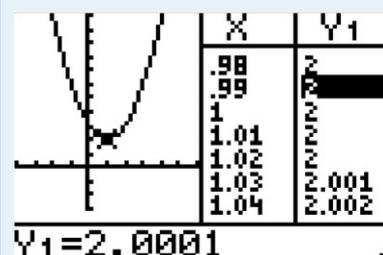
Press **2nd** **TABLE** to return to the graph and table screen.



Press **▶** to move to the column containing y -values. This shows greater precision in the box below the table.

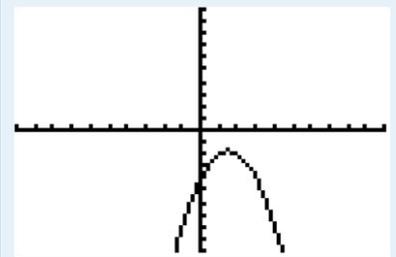


The table shows that the function has larger values at points around (1, 2). We can conclude that this is a local minimum on the curve.

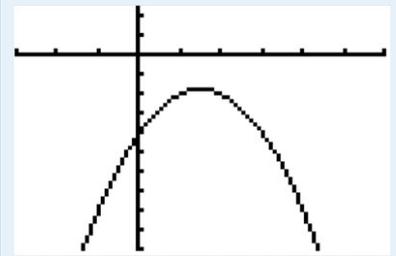


▶ Continued on next page

The calculator displays the curve with the default axes.



Adjust the window to make the quadratic curve fit the screen better.

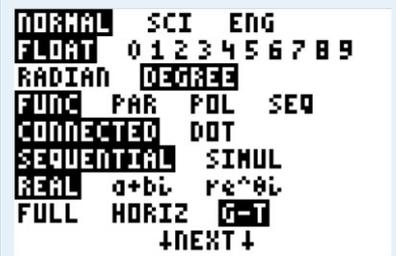


Method 1 - using a table

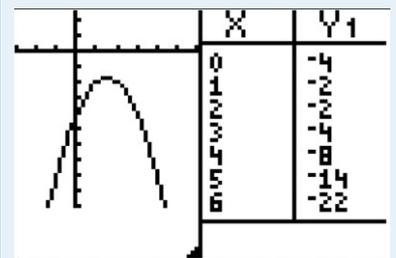
You can look at the graph and a table of the values on the graph by using a split screen.

Press **MODE** and select G-T.

Press **GRAPH**.



The maximum value shown in the table is -2 when $x = 1$ and $x = 2$.



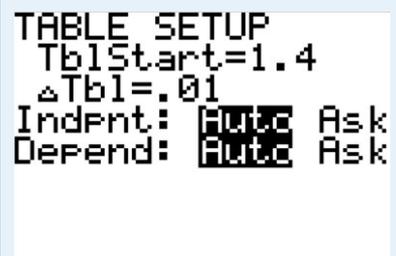
Look more closely at the values of the function between $x = 1$ and $x = 2$.

Change the settings in the table: Press **2nd** **TBLSET**.

Set TblStart to 1.4

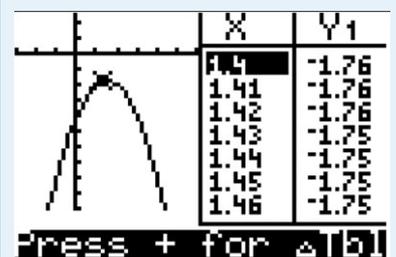
Δ Tbl to 0.01

Press **2nd** **TABLE** to return to the graph and table screen.

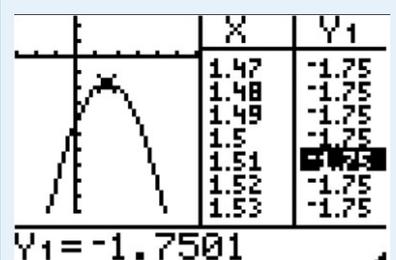


Press **▶** to move to the column containing y-values. This shows greater precision in the box below the table.

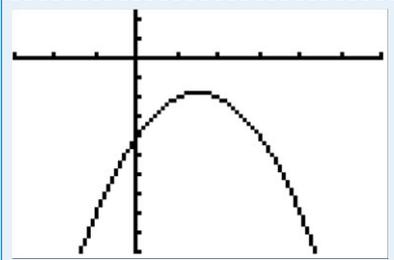
Press **▼** to scroll down until you find the maximum value of y .



The table shows that the function has smaller values at points around $(1.5, -1.75)$. We can conclude that this is a local maximum on the curve.



▶ Continued on next page

Method 2 - Using the maximum function

Press **2nd** **CALC** | 4:maximum.

Press **ENTER**.

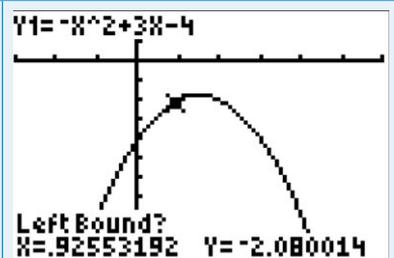
```

MATH>
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx
  
```

To find the maximum point you need to give the left and right bounds of a region that includes it.

The calculator shows a point and asks you to set the left bound. Move the point using the **◀** and **▶** keys to choose a position to the left of the maximum.

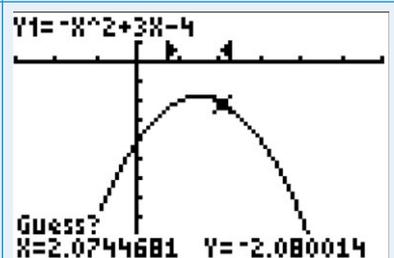
Press **ENTER**.



The calculator shows another point and asks you to set the right bound.

Move the point using the **◀** and **▶** keys so that the region between the left and right bounds contains the minimum.

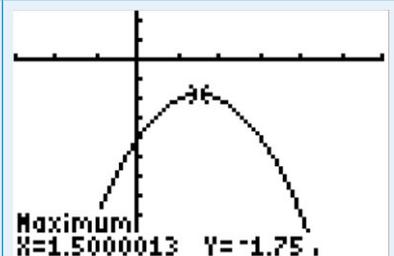
When the region contains the minimum press **ENTER**.



Press **ENTER** again to supply a guess for the value of the minimum.

The calculator displays the maximum point on the curve at (1.5, -1.75).

In this example the value of x is not exactly 1.5. This is due to the way the calculator finds the point. You should ignore small errors like this when you write down its coordinates.



Exponential functions

1.9 Drawing an exponential graph

Example 10

Draw the graph of $y = 3^x + 2$.

Press **Y=** to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

Type $3^x + 2$ and press **ENTER**.

(**Note:** Type **3** **^** **X,T,θ,n** **▶** to enter 3^x . The **▶** returns you to the baseline from the exponent.)

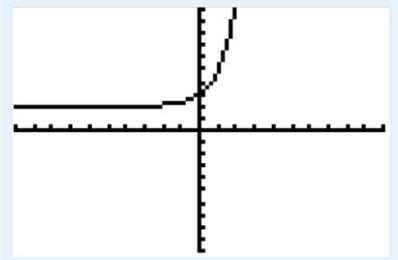
Press **ZOOM** | 6:ZStandard to use the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

```

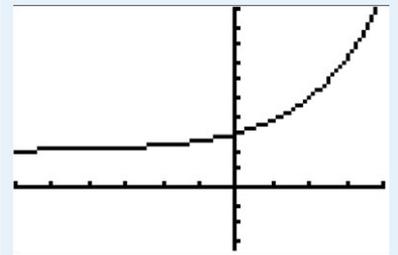
Plot1 Plot2 Plot3
Y1=3^X+2
Y2=
Y3=
Y4=
Y5=
Y6=
  
```

▶ Continued on next page

The calculator displays the curve with the default axes.



Adjust the window to make the exponential curve fit the screen better.

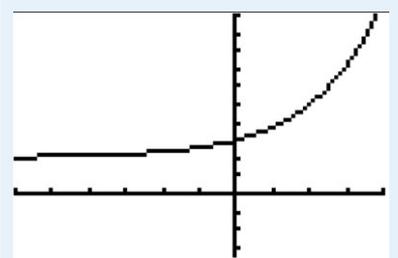


1.10 Finding a horizontal asymptote

Example 11

Find the horizontal asymptote to the graph of $y = 3^x + 2$.

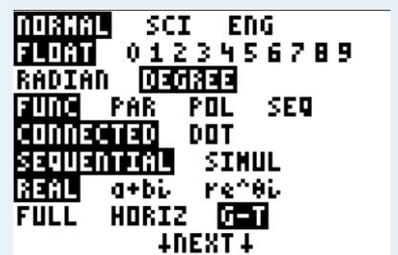
Draw the graph of $y = 3^x + 2$ (see Example 22).



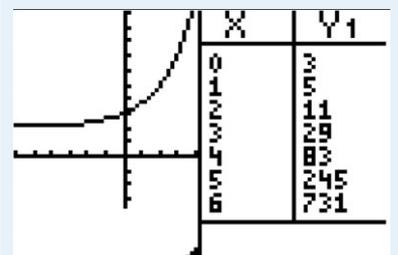
You can look at the graph and a table of the values on the graph by using a split screen.

Press **MODE** and select G-T.

Press **GRAPH**.



The values of the function are clearly decreasing as $x \rightarrow 0$.



Press **2nd** **TABLE** to switch to the table.

Press **▲** to scroll up the table.

The table shows that as the values of x get smaller, Y_1 approaches 2.



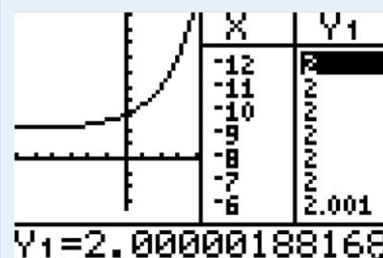
► Continued on next page

Eventually the value of Y_1 displayed in the table reaches 2.

Press \blacktriangleright to move to the column containing y -values. This shows greater precision in the box below the table. You can see, at the bottom of the screen, that the actual value of Y_1 is 2.00000188168...

We can say that $Y_1 \rightarrow 2$ as $x \rightarrow -\infty$.

The line $x = 2$ is a horizontal asymptote to the curve $y = 3^x + 2$.



Logarithmic functions

1.11 Evaluating logarithms

Example 12

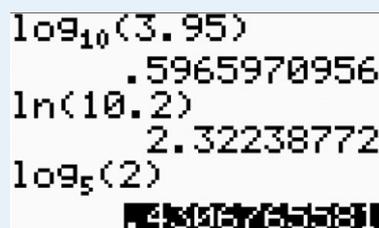
Evaluate $\log_{10} 3.95$, $\ln 10.2$ and $\log_5 2$.

Press ALPHA F2 | 5:logBASE(to open the log template. Enter the base and the argument then press ENTER .



For natural logarithms it is possible to use the same method, with the base equal to e , but it is quicker to press LN .

Note that the GDC will evaluate logarithms with any base without having to use the change of base formula.



1.12 Finding an inverse function

The inverse of a function can be found by interchanging the x and y values. Geometrically this can be done by reflecting points in the line $y = x$.

Example 13

Show that the inverse of the function $y = 10^x$ is $y = \log_{10} x$ by reflecting $y = 10^x$ in the line $y = x$.

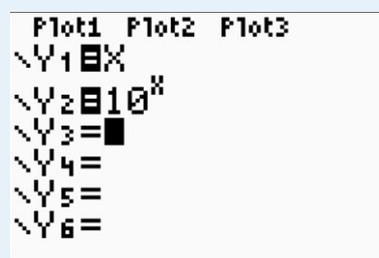
Draw the line $y = x$ so that it can be recognised as the axis of reflection.

Press Y= to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

Type x and press ENTER .

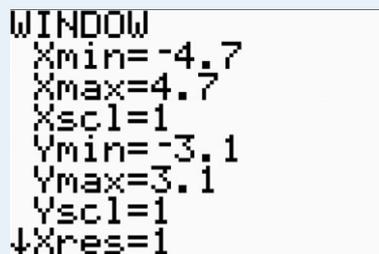
Type 10^x and press ENTER .

Note: Type 1 0 ^ $\text{X,T,}\theta,n$ \blacktriangleright to enter 10^x . The \blacktriangleright returns you to the baseline from the exponent.

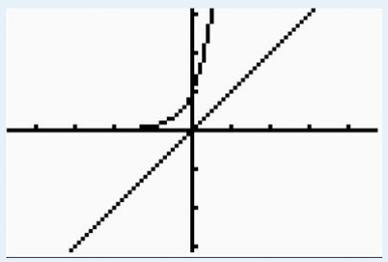
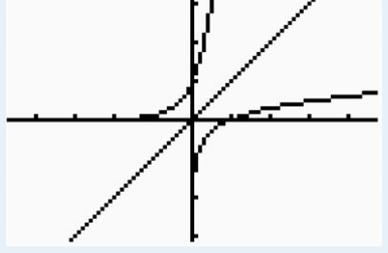
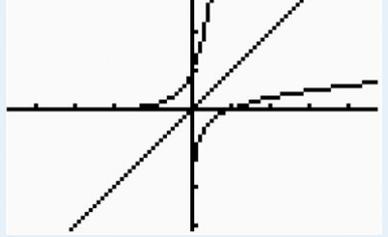


Press WINDOW and choose options as shown.

This will set up square axes $-4.7 \leq x \leq 4.7$ and $-3.1 \leq y \leq 3.1$. with the same horizontal and vertical scales.



\blacktriangleright Continued on next page

<p>Press GRAPH.</p> <p>The graphs of $y = x$ and $y = 10^x$ are displayed.</p>	
<p>Press 2nd DRAW 8:DrawInv.</p> <p>Then press ALPHA F4 and choose Y_2.</p> <p>Press ENTER.</p> <div style="border: 1px solid orange; padding: 5px; margin: 10px auto; width: fit-content;"> <p>Alternatively press LOG X,T,θ,n to enter $\log(x)$. LOG is a shorter way to enter \log_{10}.</p> </div>	<p>DrawInv Y_2</p> <p style="text-align: right;">Done</p>
<p>Press GRAPH.</p> <p>The graphs are displayed.</p> <p>The calculator will display the inverse of the function $y = 10^x$.</p>	
<p>Press Y = to display the Y= editor.</p> <p>Type $\log(x)$.</p> <div style="border: 1px solid orange; padding: 5px; margin: 10px auto; width: fit-content;"> <p>Press LOG X,T,θ,n to enter $\log(x)$. LOG is a shorter way to enter \log_{10}.</p> </div>	<pre> Plot1 Plot2 Plot3 Y1= X Y2= 10^X Y3= log(X) Y4= Y5= Y6= </pre>
<p>Press GRAPH to display the graphs of $y = x$, $y = 10^x$ and $y = \log_{10}x$.</p> <p>The inverse function and the logarithmic function coincide, showing that $y = \log_{10}x$ is inverse of the function $y = 10^x$.</p>	

1.13 Drawing a logarithmic graph

Example 14

<p>Draw the graph of $y = 2\log_{10}x + 3$.</p>	
<p>Press Y= to display the Y= editor. The default graph type is Function, so the form Y= is displayed.</p> <p>Press ALPHA F2 5:logBASE(to open the log template.</p> <p>Enter the base and the argument then press ENTER.</p>	

▶ Continued on next page

Type $2\log_{10}(x) + 3$ and press **ENTER**.

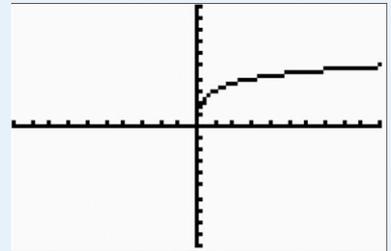
Press **ZOOM** 6: XStandard so that the calculator displays the curve with the default axes.

```

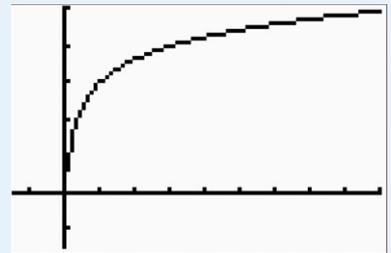
Plot1 Plot2 Plot3
Y1=2log10(X)+3
Y2=
Y3=
Y4=
Y5=
Y6=

```

The calculator displays the curve with the default axes.



Change the axes to make the logarithmic curve fit the screen better.



Trigonometric functions

1.14 Degrees and radians

Work in trigonometry will be carried out either in degrees or radians. It is important, therefore, to be able to check which mode the calculator is in and to be able to switch back and forth

Example 15

Change angle settings from radians to degrees and from degrees to radians.

Press **MODE**.

Select either RADIAN or DEGREE using the **▶** **◀** **▲** **▼** keys.

Press **ENTER**.

Press **2nd** **QUIT**.

```

NORMAL SCI ENG
FLOAT 0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
FUNC PAR POL SEQ
CONNECTED DOT
SEQUENTIAL SIMUL
REAL a+bi r e^θi
FULL HORIZ G-T
↓NEXT↓

```

1.15 Drawing a trigonometric graph

Example 16

Draw the graph of $y = 2\sin\left(x + \frac{\pi}{6}\right) + 1$.

Press $\boxed{Y=}$ to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

Type $y = 2\sin\left(x + \frac{\pi}{6}\right) + 1$ and press $\boxed{\text{ENTER}}$.

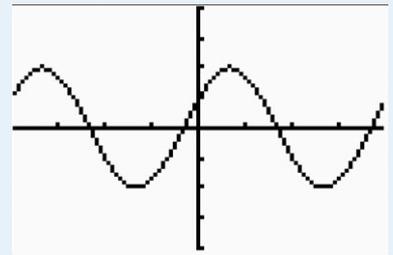
```

Plot1 Plot2 Plot3
\Y1=2sin(X+π/6)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
  
```

Press $\boxed{\text{ZOOM}} \boxed{7}$:ZTrig.

The default axes are $-6.15 \leq x \leq 6.15$ and $-4 \leq y \leq 4$.

The notation \sin^2x , \cos^2x , \tan^2x , ... is a mathematical convention that has little algebraic meaning. To enter these functions on the GDC, you should enter $(\sin(x))^2$, etc. However, the calculator will conveniently interpret $\sin(x)^2$ as $(\sin(x))^2$.



More complicated functions

1.16 Solving a combined quadratic and exponential equation

Follow the same GDC procedure when solving simultaneous equations graphically and solving a combined quadratic and exponential equation. See Examples 18 and 24.

Example 17

Solve the equation $x^2 - 2x + 3 = 3 \cdot 2^{-x} + 4$

To solve the equation, find the point of intersection between the quadratic function $y_1 = x^2 - 2x + 3$ and the exponential function $y_2 = 3 \times 2^{-x} + 3$.

Press $\boxed{Y=}$ to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

Type $x^2 - 2x + 3$ in Y_1 and press $\boxed{\text{ENTER}}$. Then type $3 \times 2^{-x} + 4$ in Y_2 and press $\boxed{\text{ENTER}}$.

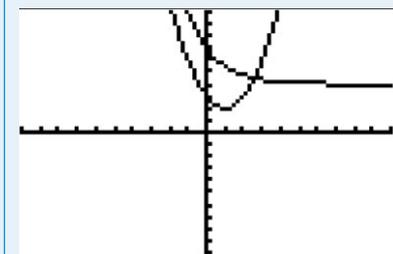
(**Note:** Type $\boxed{2} \boxed{\wedge} \boxed{(-)} \boxed{x,T,\theta,n} \boxed{\blacktriangleright}$ to enter 2^{-x} . The \blacktriangleright returns you to the baseline from the exponent.)

```

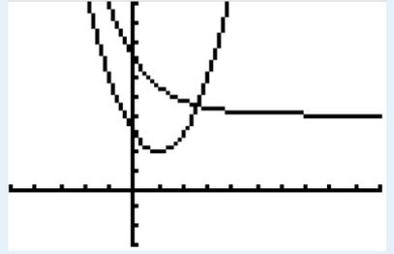
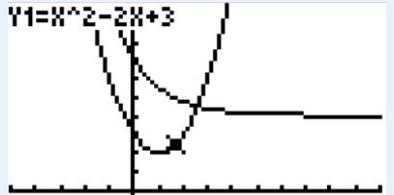
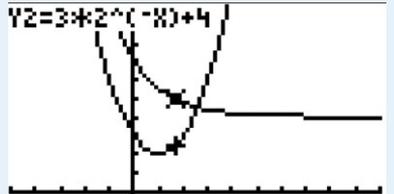
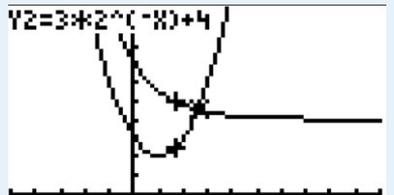
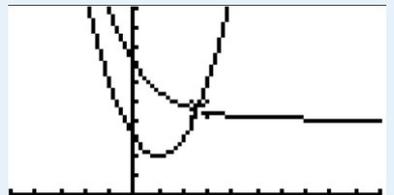
Plot1 Plot2 Plot3
\Y1=X^2-2X+3
\Y2=3*2^-X+4
\Y3=
\Y4=
\Y5=
\Y6=
  
```

Press $\boxed{\text{ZOOM}} \boxed{|} \boxed{6}$:Z Standard to use the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

The calculator displays the curves with the default axes.



▶ Continued on next page

Adjust the window to make the quadratic curve fit the screen better.	
Press 2nd CALC 5:intersect. Press ENTER .	<pre>CALCULATE 1:value 2:zero 3:minimum 4:maximum 5:intersect 6:dy/dx 7:∫f(x)dx</pre>
Press ENTER to select the first curve.	<pre>Y1=X^2-2X+3  First curve? X=1.7021277 Y=2.4929833</pre>
Press ENTER to select the second curve.	<pre>Y2=3*2^(-X)+4  Second curve? X=1.7021277 Y=4.9219976</pre>
Select a point close to the intersection using the ◀ and ▶ keys and press ENTER .	<pre>Y2=3*2^(-X)+4  Guess? X=2.6595745 Y=4.4747987</pre>
The calculator displays the intersection of the two straight lines at the point (2.58, 4.50). The solutions are $x = 2.58$ and $y = 4.50$.	<pre>Intersection X=2.5815169 Y=4.5011956 </pre>

1.17 Using sinusoidal regression

Example 18

It is known that the following data can be modelled using a sine curve:

x	0	1	2	3	4	5	6	7
y	6.9	9.4	7.9	6.7	9.2	8.3	6.5	8.9

Use sine regression to find a function to model this data.

Press **STAT** | 1:Edit and press **F3**.

Type the x -values in the first column (L1) and the y -values in the second column (L2).

Press **ENTER** or **↓** after each number to move down to the next cell.

Press **▶** to move to the next column.

You can use columns from L1 to L6 to enter the lists.

L1	L2	L3	1
0	6.9	-----	
1	9.4		
2	7.9		
3	6.7		
4	9.2		
5	8.3		
6	6.5		
7	8.9		

L1()=0

Press **2nd** **STAT PLOT** and **eto** select Plot1.

Select On, choose the scatter diagram option, Xlist as L1 and Ylist as L2.

You can choose one of the three types of mark.

```

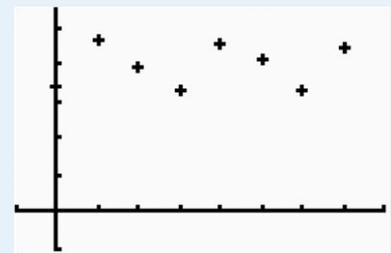
STAT PLOTS
1:Plot1...Off
  L1 L2
2:Plot2...Off
  L1 L2
3:Plot3...Off
  L1 L2
4↓PlotsOff

3:Plot1 Plot2 Plot3
Off Off
Type: [Scatter] [Line] [Bar]
      [None] [Dot] [Cross]
Xlist:L1
Ylist:L2
Mark: [ ] [ ] [ ]
  
```

Press **ZOOM** 9:ZoomStat.

Adjust your window settings to show your data and the x - and y -axes.

You now have a scatter plot of x against y .



Press **2nd** **↑** to return to the Home screen.

Press **STAT** **CALC** | C:SinReg.

Press **2nd** **L1** , **2nd** **α**, **ALPHA** **F4** choose Y_1 and press **F3**.

Press **F3** again.

```

SinReg L1,L2,Y1
  
```

On screen, you will see the result of the sinusoidal regression.

The equation is in the form $y = a\sin(bx + c) + d$ and you will see the values of a , b , c and d displayed separately.

The equation of the sinusoidal regression line is

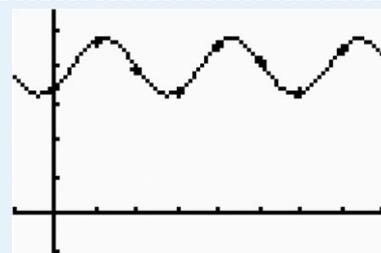
$$y = 1.51\sin(2.00x - 0.80) + 7.99$$

```

SinReg
y=a*sin(bx+c)+d
a=1.506000561
b=2.002900961
c=-.7998734807
d=7.991078656
  
```

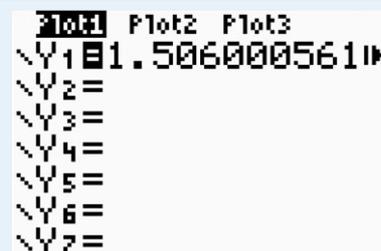
▶ Continued on next page

Press **GRAPH** to return to the Graphs page.



Press **▸**.

The regression line is now shown in Y_1 . You can see the full equation if you scroll to the right.



2 Differential calculus

Finding gradients, tangents and maximum and minimum points

2.1 Finding the gradient at a point

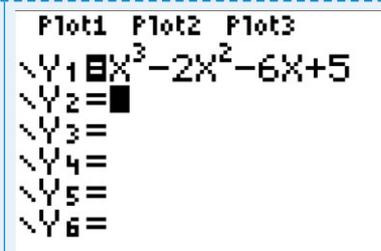
Example 19

Find the gradient of the cubic function $y = x^3 - 2x^2 - 6x + 5$ at the point where $x = 1.5$.

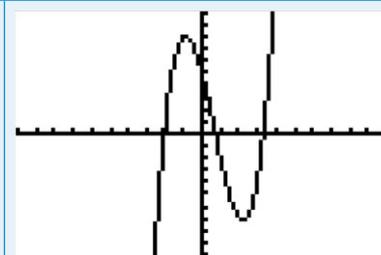
Press **Y=** to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

Type $y = x^3 - 2x^2 - 6x + 5$ and press **ENTER**.

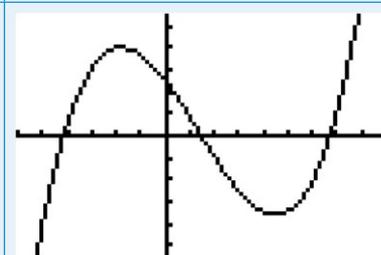
(Note: Type **x,T,θ,n** **^** **3** **▸** to enter x^3 . The **▸** returns you to the baseline from the exponent.)



Press **ZOOM** **|** **6:ZStandard** to use the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.



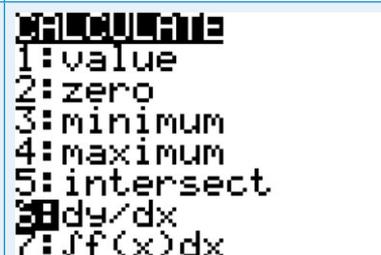
Adjust the window to make the cubic curve fit the screen better.



Press **2nd** **CALC** **|** **6:dy/dx**.

Press **ENTER**.

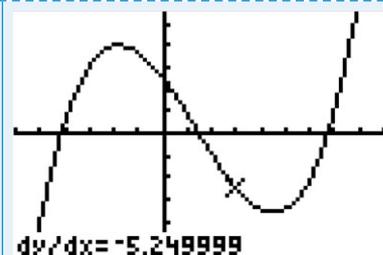
Press **1** **.** **5** **ENTER**.



▶ Continued on next page

The calculator displays the gradient of the curve at the point where $x = 1.5$. The gradient is -5.25 .

In this example the value of dy/dx is not exactly -5.25 . This is due to the way the calculator finds the point gradient. You should ignore small errors like this when you write down the coordinates of a gradient at a the point.

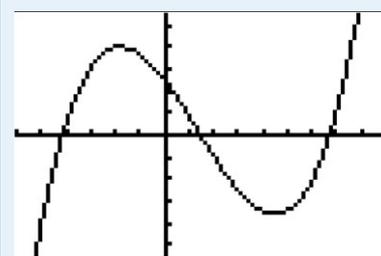


2.2 Drawing a tangent to a curve

Example 20

Draw a tangent to the curve $y = x^3 - 2x^2 - 6x + 5$ where $x = -0.5$.

First draw the graph of $y = x^3 - 2x^2 - 6x + 5$ (see Example 30).



Press **2nd** **DRAW**.
Choose 5:Tangent.
Press **ENTER**.

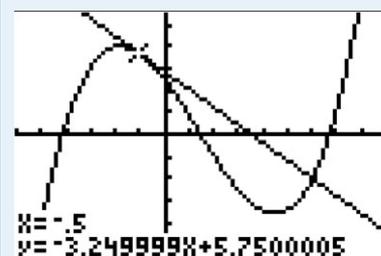
```

DRAW POINTS STO
1:ClrDraw
2:Line(
3:Horizontal
4:Vertical
5:Tangent(
6:DrawF
7:Shade(
  
```

Press **(-)** **0** **.** **5** **ENTER**.

The equation of the tangent is
 $y = -3.25x + 5.75$

In this example the values -3.25 and 5.75 are not shown as being exact. This is due to the way the calculator finds the values. You should ignore small errors like this when you write down the equation of a tangent.

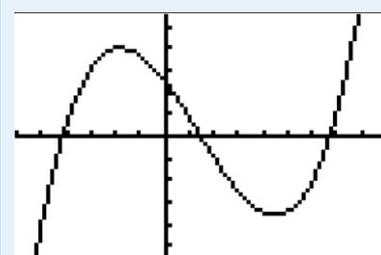


2.3 Finding maximum and minimum points

Example 21

Find the local maximum and local minimum points on the cubic curve.

First draw the graph of $y = x^3 - 2x^2 - 6x + 5$ (see Example 30).



▶ Continued on next page

Press **2nd** **CALC** | 3:minimum.

Press **ENTER**.

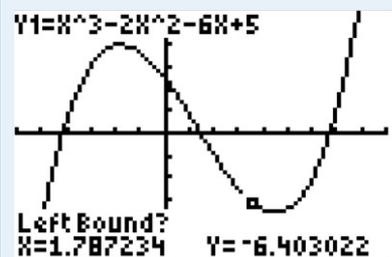
```

CALC
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx
  
```

To find the minimum point you need to give the left and right bounds of a region that includes it.

The calculator shows a point and asks you to set the left bound. Move the point using the **◀** and **▶** keys to choose a position to the left of the minimum.

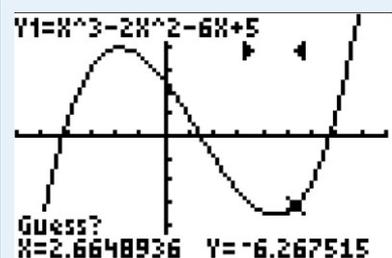
Press **ENTER**.



The calculator shows another point and asks you to set the right bound.

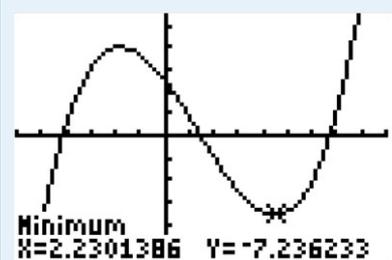
Move the point using the **◀** and **▶** keys so that the region between the left and right bounds contains the minimum.

When the region contains the minimum press **ENTER**.



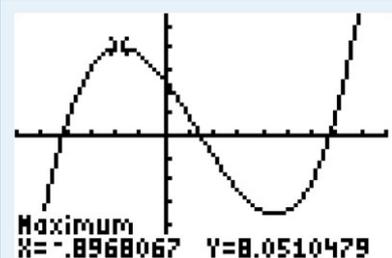
Press **ENTER** again to supply a guess for the value of the minimum.

The calculator displays the local minimum at the point (2.23, -7.24).



Press **2nd** **CALC** | 3:maximum **ENTER**. To find the local maximum point on the curve in exactly the same way.

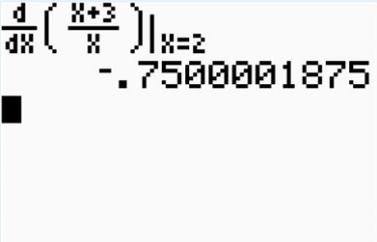
The maximum point is (-0.897, 8.05).



2.4 Finding a numerical derivative

Using the calculator it is possible to find the numerical value of any derivative for any value of x . The calculator will not, however, differentiate a function algebraically. This is equivalent to finding the gradient at a point graphically.

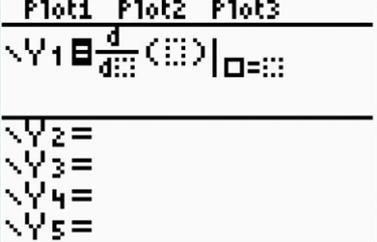
Example 22

<p>If $y = \frac{x+3}{x}$, evaluate $\frac{dy}{dx} \Big _{x=2}$</p>	
<p>Press ALPHA F2.</p> <p>Choose 3: nDeriv(to choose the derivative template.</p>	 <pre> 1: abs(2: ∫(3: nDeriv(4: fnInt(5: 10BASEC FRAC FUNC NTRX YVAR </pre>
<p>Enter x and the function in the template. Enter the value 2.</p> <p>Press ENTER.</p>	 <pre> d d/dx () □=:: </pre>
<p>The calculator shows that the value of the first derivative of $y = \frac{x+3}{x}$ is -0.75 when $x = 2$.</p>	 <pre> d d/dx ((x+3)/x) x=2 -.7500001875 </pre>

2.5 Graphing a numerical derivative

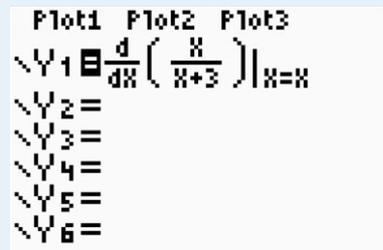
Although the calculator can only evaluate a numerical derivative at a point, it will graph the gradient function for all values of x .

Example 23

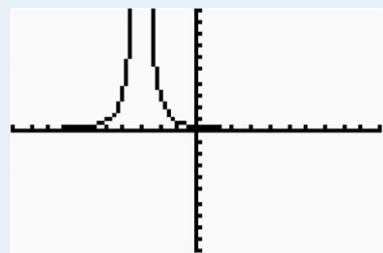
<p>If $y = \frac{x}{x+3}$, draw the graph of $\frac{dy}{dx}$.</p>	
<p>Press Y= to display the Y= editor. The default graph type is Function, so the form Y= is displayed.</p>	 <pre> P1ot1 P1ot2 P1ot3 \Y1 = \Y2 = \Y3 = \Y4 = \Y5 = \Y6 = \Y7 = </pre>
<p>Press ALPHA F2.</p> <p>Choose 3: nDeriv(to choose the derivative template.</p>	 <pre> P1ot1 P1ot2 P1ot3 \Y1 d d/dx () □=:: \Y2 = \Y3 = \Y4 = \Y5 = </pre>

▶ Continued on next page

In the template enter x , the function $\frac{x}{x+3}$ and the value x .
 Press **ENTER**.



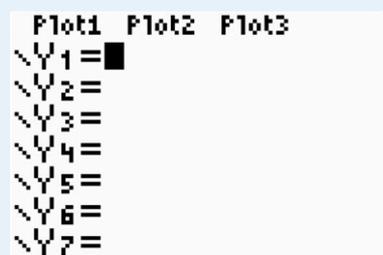
Press **ZOOM** 6:ZStandard.
 The calculator displays the graph of the numerical derivative function of $y = \frac{x}{x+3}$.



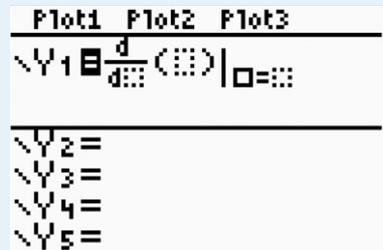
Example 24

Find the values of x on the curve $y = \frac{x^3}{3} + x^2 - 5x + 1$ where the gradient is 3.

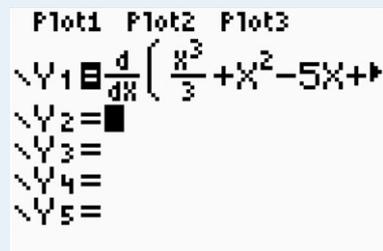
Press **Y=** to display the Y= editor. The default graph type is Function, so the form Y= is displayed.



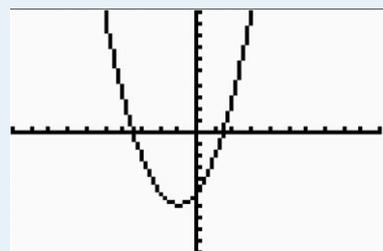
Press **ALPHA** **F2**.
 Choose 3: nDeriv(to choose the derivative template.



In the template enter x , the function $\frac{x^3}{3} + x^2 - 5x + 1$ and the value x .
 Press **ENTER**.

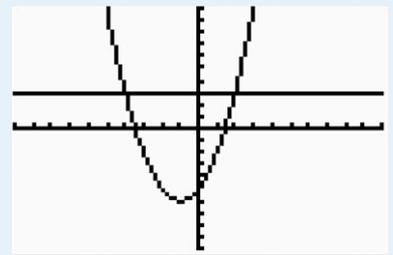


Press **ZOOM** 6:ZStandard.
 The calculator displays the graph of the numerical derivative function of $y = \frac{x^3}{3} + x^2 - 5x + 1$.



▶ Continued on next page

Press $Y=$ to display the Y= editor.
 Enter the function $Y_2 = 3$.
 Press GRAPH .
 The calculator now displays the curve and the line $y = 3$.



To find the points of intersection between the curve and the line.
 Press 2nd CALC | 5:intersect.
 Press ENTER .

```

CALCULATE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx
  
```

Press ENTER to select the first curve.

```

Y1=nderiv((X^3)+3+X^2-5X
First curve?
X=0      Y=-5
  
```

Press ENTER to select the second curve.

```

Y2=3
Second curve?
X=0      Y=3
  
```

Select a point close to the intersection using the \leftarrow and \rightarrow keys
 and press ENTER .
 Repeat for the second point of intersection.

```

Y2=3
Guess?
X=1.7021277  Y=3
  
```

The curve has gradient 3 when $x = -4$ and $x = 2$

In this example the value of x is not exactly 2. This is due to the way the calculator finds the point. You should ignore small errors like this when you write down the coordinates of a gradient at a point.

```

Intersection
X=1.9999999  Y=3
  
```

3 Integral calculus

The calculator can find the values of definite integrals either on a calculator page or graphically. The calculator method is quicker, but the graphical method is clearer and shows discontinuities, negative areas and other anomalies that can arise.

3.1 Finding the value of an indefinite integral

Example 25

Evaluate $\int_2^8 \left(x - \frac{3}{\sqrt{x}} \right) dx$.

Press **ALPHA** **F2**.

Choose 4: fnInt(to choose the integral template.

In this example you will also use templates to enter the rational function and the square root.

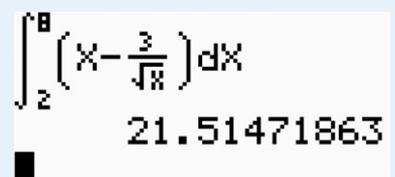


Enter the upper and lower limits, the function and x in the template.

Press **ENTER**.



The value of the integral is 21.5 (to 3 sf).



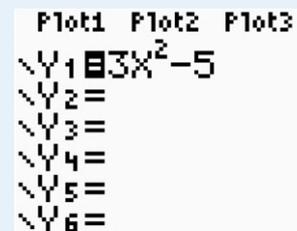
3.2 Finding the area under a curve

Example 26

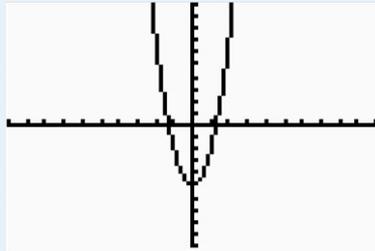
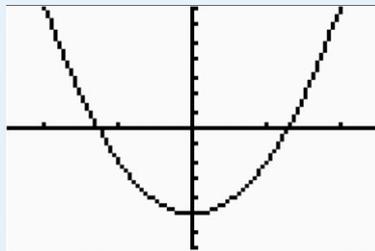
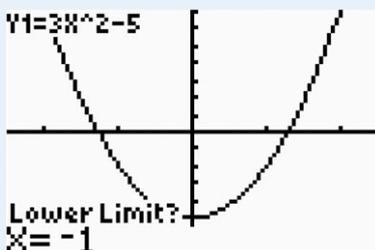
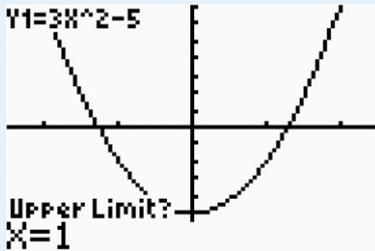
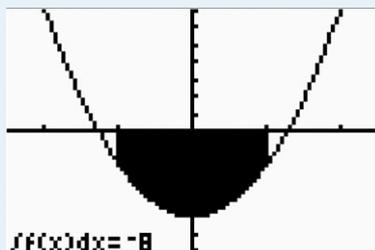
Find the area bounded by the curve $y = 3x^2 - 5$, the x -axis and the lines $x = -1$ and $x = 1$.

Press **Y=** to display the Y= editor. The default graph type is Function, so the form $Y_1 =$ is displayed.

Type $y = 3x^2 - 5$ and press **ENTER**.



▶ Continued on next page

<p>Press ZOOM 6:ZStandard. The default axes are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.</p>	
<p>Adjust the window settings to view the curve better.</p>	
<p>Press 2nd CALC 7:∫f(x)dx. The calculator prompts you to enter the lower limit for the integral. Type -1 and press ENTER. Be sure to use the (-) key.</p>	
<p>Type 1 and press ENTER.</p>	
<p>The area found is shaded and the value of the integral (-8) is shown on the screen. Note: since the area lies below the x-axis in this case, the integral is negative. The required area is 8.</p>	

4 Vectors

Scalar product

4.1 Calculating a scalar product

Example 27

There is no scalar product function on the TI-84 plus, but you can find the result by multiplying the vectors as lists and then finding the sum of the terms in the list.

Evaluate the scalar products:

$$\mathbf{a} \begin{pmatrix} 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 4 \end{pmatrix} \quad \mathbf{b} \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$$

a Press **2nd** LIST | MATH | 5:sum(.

```
NAMES OPS MATH
1:min(
2:max(
3:mean(
4:median(
5:sum(
6:Prod(
7↓stdDev(
```

Enter the vectors as lists using curly brackets { }. Separate the terms of the vectors using commas.
Multiply the two vector lists together.

```
4m( (1,3)*(-3,4)
```

Close the bracket and press **ENTER**.

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 4 \end{pmatrix} = 9$$

```
sum( (1,3)*(-3,4)
9
```

b Press **2nd** LIST | MATH | 5:sum(.

```
NAMES OPS MATH
1:min(
2:max(
3:mean(
4:median(
5:sum(
6:Prod(
7↓stdDev(
```

▶ Continued on next page

Enter the vectors as lists using curly brackets { }. Separate the terms of the vectors using commas.
Multiply the two vector lists together.

```
{1, -1, 4} * {3, 2, -1}
```

Close the bracket and press **ENTER**.

$$\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} = -3$$

```
SUM({1, -1, 4} * {3, 2, -1})
```

4.2 Calculating the angle between two vectors

The angle θ between two vectors \vec{a} and \vec{b} , can be calculated using the formula

$$\theta = \arccos\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}\right)$$

Example 28

Calculate the angle between $2\vec{i} + 3\vec{j}$ and $3\vec{i} - \vec{j}$

Press **MODE**.

Select either RADIAN or DEGREE (according to the units you need your answer in) using the **▶** **◀** **▲** **▼** keys.

Press **ENTER**.

Press **2nd** **QUIT**.

```
NORMAL SCI ENG
FLOAT 0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
FUNC PAR POL SEQ
CONNECTED DOT
SEQUENTIAL SIMUL
REAL a+bi re^θi
FULL HORIZ G-T
↓NEXT↓
```

Press **2nd** **DISTR**.

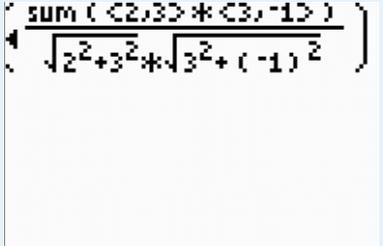
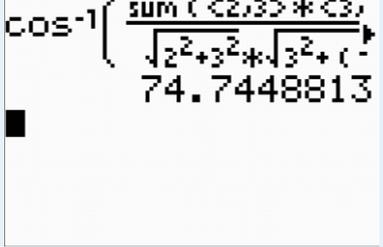
```
cos-1(
```

Press **ALPHA** **F1** and select the fraction template 1:n/d

```
cos-1(
```

```
1: n/d
2: Un/d
3: n/d d Un/d
4: F d D
FRAC FUNC MTRX VVAR
```

▶ Continued on next page

<p>Press 2nd LIST MATH 5:sum(.</p>	
<p>Enter the vectors as lists using curly brackets { }. Separate the terms of the vectors using commas. Multiply the two vector lists together. To calculate the magnitudes of the vectors use the formula $\vec{a}\vec{i} + \vec{b}\vec{j} = \sqrt{a^2 + b^2}$ Use the ▸ key to exit the templates before entering the final bracket.</p>	
<p>Press ENTER. The angle between $2\vec{i} + 3\vec{j}$ and $3\vec{i} - \vec{j}$ is 74.7°.</p>	

5 Statistics and probability

You can use your GDC to draw charts to represent data and to calculate basic statistics such as mean, median, etc. Before you do this you need to enter the data in a list.

Entering data

There are two ways of entering data: as a list or as a frequency table.

5.1 Entering lists of data

Example 29

<p>Enter the data in the list: 1, 1, 3, 9, 2.</p>																														
<p>Press STAT 1: Edit and press ENTER. Type the numbers in the first column (L1). Press ENTER or ▾ after each number to move down to the next cell. L1 will be used later when you want to make a chart or to do some calculations with this data. You can use columns from L1 to L6 to enter the list.</p>		<table border="1" data-bbox="1093 1579 1476 1825"> <thead> <tr> <th>L1</th> <th>L2</th> <th>L3</th> <th>1</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>-----</td> <td>-----</td> <td></td> </tr> <tr> <td>1</td> <td></td> <td></td> <td></td> </tr> <tr> <td>3</td> <td></td> <td></td> <td></td> </tr> <tr> <td>9</td> <td></td> <td></td> <td></td> </tr> <tr> <td>2</td> <td></td> <td></td> <td></td> </tr> <tr> <td colspan="4">L1(6)=</td> </tr> </tbody> </table>	L1	L2	L3	1	1	-----	-----		1				3				9				2				L1(6)=			
L1	L2	L3	1																											
1	-----	-----																												
1																														
3																														
9																														
2																														
L1(6)=																														

5.2 Entering data from a frequency table

Example 30

Enter the data in the table:	Number	1	2	3	4	5
	Frequency	3	4	6	5	2

Press **STAT** | 1:Edit and press **ENTER**.

Type the numbers in the first column (L1) and the frequencies in the second column (L2).

Press **ENTER** or \blacktriangledown after each number to move down to the next cell.

Press \blacktriangleright to move to the next column.

L1 and L2 will be used later when you want to make a chart or to do some calculations with this data. You can use columns from L1 to L6 to enter the lists.

CALC TESTS

1:Edit...

2:SortA(

3:SortD(

4:ClrList

5:SetUpEditor

L1	L2	L3	2
1	3		
2	4		
3	6		
4	5		
5	2		

L2(6) =

Drawing charts

Charts can be drawn from a list or from a frequency table.

5.3 Drawing a frequency histogram from a list

Example 31

Draw a frequency histogram for this data: 1, 1, 3, 9, 2.

Enter the data in L1 (see Example 5).

Press **2nd** **STAT PLOT** and **ENTER** to select Plot1.

Select On, choose the histogram option and leave Xlist as L1 and Freq as 1.

STAT PLOTS

1:Plot1...Off

2:Plot2...Off

3:Plot3...Off

4↓PlotsOff

Plot1 Plot2 Plot3

On Off

Type: \square \triangle \square

Xlist:L1

Freq:1

Press **ZOOM** | 9:Stat.

The automatic scales do not usually give the best display of the histogram. You will need to change the default values.

You may need to delete any function graphs. **Y=**

MEMORY

3↑Zoom Out

4:ZDecimal

5:ZSquare

6:ZStandard

7:ZTrig

8:ZInteger

9↓ZoomStat

Press **WINDOW** and choose options as shown.

Xmin and Xmax should include the range of the data.

Ymin and Ymax should include the maximum frequency and should go below zero.

Xscl will define the width of the bars.

WINDOW

Xmin=0

Xmax=11

Xscl=1

Ymin=-1

Ymax=3

Yscl=1

↓Xres=1

Press **TRACE**.

Use the \blacktriangleright key to move to each of the bars and display their value and frequency.

You should now see a frequency histogram for the data in the list 1, 1, 3, 9, 2.

P1:L1

min=1

max<2

n=2

5.4 Drawing a frequency histogram from a frequency table

Example 32

Draw a frequency histogram for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

Enter the data in L1 and L2 (see Example 6). Press **2nd** **STAT PLOT** and **ENTER** to select Plot 1. Select On, choose the histogram option and leave XList as L1 and Freq as L2.

```

STAT PLOTS
1:Plot1...Off
  L1 L2
2:Plot2...Off
  L1 L2
3:Plot3...Off
  L1 L2
4↓PlotsOff
                    
```

```

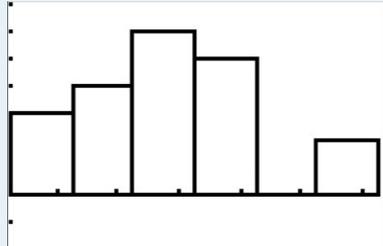
Plot1 Plot2 Plot3
Off Off
Type:
Xlist:L1
Freq:L2
                    
```


Press **ZOOM** | 9:Stat. The automatic scales do not usually give the best display of the histogram. You will need to change the default values.

You may need to delete any function graphs. **Y=**

```

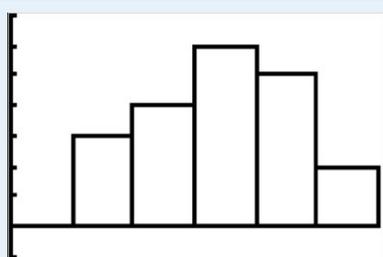
MEMORY
3↑Zoom Out
4:ZDecimal
5:ZSquare
6:ZStandard
7:ZTrig
8:ZInteger
9↓ZoomStat
                    
```



Press **WINDOW** and choose options as shown. Xmin and Xmax should include the range of the data. Ymin and Ymax should include the maximum frequency and should go below zero. Xscl will define the width of the bars.

```

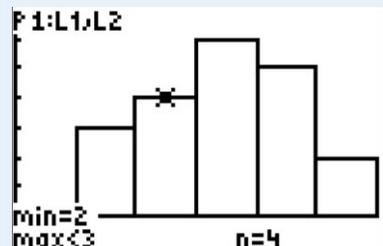
WINDOW
Xmin=0
Xmax=6
Xscl=1
Ymin=-1
Ymax=7
Yscl=1
↓Xres=1
                    
```



Press **TRACE**. Use the **▶** key to move to each of the bars and display their value and frequency.

```

F1:L1,L2
min=2
max=3
n=4
                    
```



You should now see a frequency histogram for the data in the list 1, 1, 3, 9, 2.

5.5 Drawing a box and whisker diagram from a list

Example 33

Draw a box and whisker diagram for this data:
1, 1, 3, 9, 2.

Enter the data in L1 (see Example 5). Press **2nd** **STAT PLOT** and **ENTER** to select Plot 1. Select On, choose the box and whisker option and leave XList as L1 and Freq as 1.

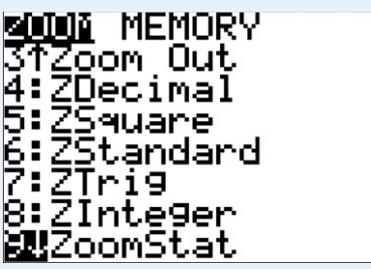
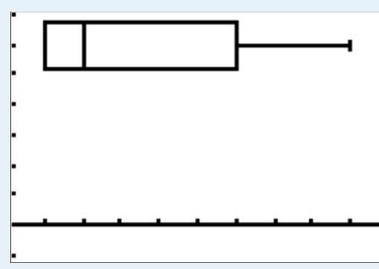
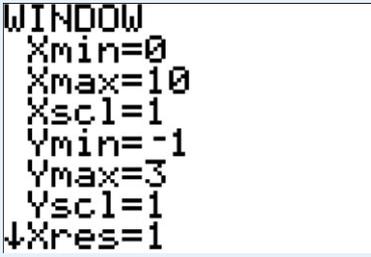
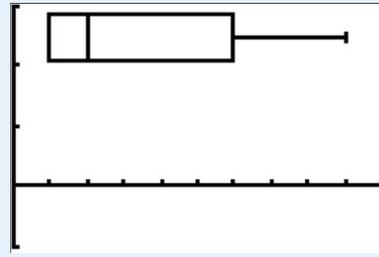
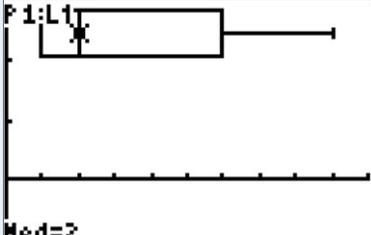
```

STAT PLOTS
1:Plot1...Off
  L1 L2
2:Plot2...Off
  L1 L2
3:Plot3...Off
  L1 L2
4↓PlotsOff
                    
```

```

Plot1 Plot2 Plot3
On Off
Type:
Xlist:L1
Freq:1
                    
```

▶ Continued on next page

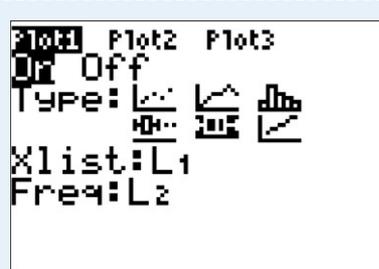
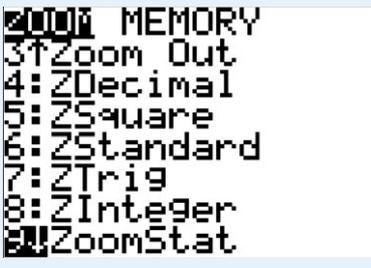
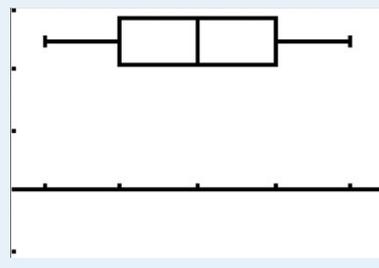
<p>Press ZOOM 9:Stat.</p> <p>The automatic scales do not usually give the best display of the box and whisker diagram. You will need to change the default values.</p> <div style="border: 1px solid orange; padding: 5px; width: fit-content; margin-left: auto; margin-right: auto;"> <p>You may need to delete any function graphs. Y=</p> </div>		
<p>Press WINDOW and choose options as shown.</p> <p>Xmin and Xmax should include the range of the data.</p> <p>Ymin and Ymax do not affect the way in which the diagram is displayed.</p>		
<p>Press TRACE.</p> <p>Use the ▶ key to move the cursor over the plot to see the quartiles, Q1 and Q3, the median and the maximum and minimum values.</p>		

5.6 Drawing a box and whisker diagram from a frequency table

Example 34

Draw a box and whisker diagram for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

<p>Enter the data in L1 and L2 (see Example 6).</p> <p>Press 2nd STAT PLOT and ENTER to select Plot 1. Select On, choose the box and whisker diagram option and leave XList as L1 and Freq as L2.</p>		
<p>Press ZOOM 9:Stat.</p> <p>The automatic scales do not usually give the best display of the box and whisker diagram. You will need to change the default values.</p> <div style="border: 1px solid orange; padding: 5px; width: fit-content; margin-left: auto; margin-right: auto;"> <p>You may need to delete any function graphs. Y=</p> </div>		

▶ Continued on next page

<p>Press WINDOW and choose options as shown. Xmin and Xmax should include the range of the data. Ymin and Ymax do not affect the way in which the diagram is displayed.</p>	<pre>WINDOW Xmin=0 Xmax=6 Xscl=1 Ymin=-1 Ymax=3 Yscl=1 ↓Xres=1</pre>	
<p>Press TRACE. Use the ▶ key to move the cursor over the plot to see the quartiles, Q1 and Q3, the median and the maximum and minimum values.</p>		

Calculating statistics

You can calculate statistics such as mean, median, etc. from a list, or from a frequency table.

5.7 Calculating statistics from a list

Example 35

<p>Calculate the summary statistics for this data: 1, 1, 3, 9, 2</p>																								
<p>Enter the data in L1 (see Example 5). Press STAT CALC 1:1-Var Stats. Type 2nd L1 and press ENTER.</p>	<pre>EDIT [DEL] TESTS 1:1-Var Stats 2:2-Var Stats 3:Med-Med 4:LinReg(ax+b) 5:QuadReg 6:CubicReg 7↓QuartReg</pre>	<pre>1-Var Stats L1</pre>																						
<p>The information shown will not fit on a single screen. You can scroll up and down to see it all. The statistics calculated for the data are:</p> <table border="0" style="width: 100%;"> <tr><td style="text-align: right;">mean</td><td>\bar{x}</td></tr> <tr><td style="text-align: right;">sum</td><td>Σx</td></tr> <tr><td style="text-align: right;">sum of squares</td><td>Σx^2</td></tr> <tr><td style="text-align: right;">sample standard deviation</td><td>s_x</td></tr> <tr><td style="text-align: right;">population standard deviation</td><td>σ_x</td></tr> <tr><td style="text-align: right;">number</td><td>n</td></tr> <tr><td style="text-align: right;">minimum value</td><td>MinX</td></tr> <tr><td style="text-align: right;">lower quartile</td><td>Q_1</td></tr> <tr><td style="text-align: right;">median</td><td>Med</td></tr> <tr><td style="text-align: right;">upper quartile</td><td>Q_3</td></tr> <tr><td style="text-align: right;">maximum value</td><td>MaxX</td></tr> </table>	mean	\bar{x}	sum	Σx	sum of squares	Σx^2	sample standard deviation	s_x	population standard deviation	σ_x	number	n	minimum value	MinX	lower quartile	Q_1	median	Med	upper quartile	Q_3	maximum value	MaxX	<pre>1-Var Stats x̄=3.2 Σx=16 Σx²=96 Sx=3.346640106 σx=2.993325909 ↓n=5</pre>	<pre>1-Var Stats ↑n=5 minX=1 Q1=1 Med=2 Q3=6 maxX=9</pre>
mean	\bar{x}																							
sum	Σx																							
sum of squares	Σx^2																							
sample standard deviation	s_x																							
population standard deviation	σ_x																							
number	n																							
minimum value	MinX																							
lower quartile	Q_1																							
median	Med																							
upper quartile	Q_3																							
maximum value	MaxX																							

5.8 Calculating statistics from a frequency table

Example 36

Calculate the summary statistics for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

Enter the data in L1 and L2 (see Example 6).

Press **STAT** | **CALC** | 1:1-Var Stats.

Type **2nd** **L1** , **2nd** **L2** and press **ENTER**.

```
1-Var Stats L1,L2
```

The information shown will not fit on a single screen. You can scroll up and down to see it all. The statistics calculated for the data are:

mean	\bar{x}
sum	Σx
sum of squares	Σx^2
sample standard deviation	s_x
population standard deviation	σ_x
number	n
minimum value	$\min X$
lower quartile	Q_1
median	Med
upper quartile	Q_3
maximum value	$\max X$

```
1-Var Stats
x̄=2.95
Σx=59
Σx²=203
Sx=1.234376041
σx=1.203120942
↓n=20
```

```
1-Var Stats
↑n=20
minX=1
Q1=2
Med=3
Q3=4
maxX=5
```

5.9 Calculating the interquartile range

Example 37

Calculate interquartile range for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

The interquartile range is the difference between the upper and lower quartiles ($Q_3 - Q_1$).

First calculate the summary statistics for this data (see Example 12).

(**Note:** The values of the summary statistics are stored after One-Variable Statistics have been calculated and remain stored until the next time they are calculated.)

Press **VARS** | 5:Statistics... | **PTS** | 9:Q3 **ENTER** **-** **VARS** | 5:Statistics... | **PTS** | 7:Q1 **ENTER**.

The calculator now displays the result:

Interquartile range = $Q_3 - Q_1 = 2$

```
Q3-Q1
2
```

5.10 Using statistics

The calculator stores the values you calculate in One-Variable Statistics so that you can access them in other calculations. These values are stored until you do another One-Variable Statistics calculation.

Example 38

Calculate the $\bar{x} + \sigma_x$ for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

First calculate the summary statistics for this data (see Example 12).

(**Note:** The values of the summary statistics are stored after One-Variable Statistics have been calculated and remain stored until the next time they are calculated.)

Press **VAR** | 5:Statistics... | 2: \bar{x} **ENTER** **-** **VAR** | 5:Statistics... 4: σ_x **ENTER**.

The calculator now displays the result:

$\bar{x} + \sigma_x = 4.15$ (to 3 sf)

```

 $\bar{x} + \sigma_x$ 
4.153120942
  
```

Calculating binomial probabilities

5.11 The use of nCr

Example 39

Find the value of $\binom{8}{3}$ (or ${}_8C_3$).

Press **8**.

Press **MATH** 3:nCr.

Press **3** **ENTER**.

```

MATH NUM CPX 1234
1:rand
2:nPr
3:nCr
4:!
5:randInt(
6:randNorm(
7:randBin(
  
```

Press **ENTER**.

```

8 nCr 3
56
  
```

Example 40

List the values of $\binom{4}{r}$ for $r = 0, 1, 2, 3, 4$.

Press Y= to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

Press 4 .

Press m 3:nCr .

Press $\text{x,t,}\theta,n \text{ ENTER}$.

```

Plot1 Plot2 Plot3
Y1=4 nCr X
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
  
```

Press 2nd TABLE .

The table shows that

$$\binom{4}{0} = 1, \binom{4}{1} = 4, \binom{4}{2} = 6, \binom{4}{3} = 4$$

$$\text{and } \binom{4}{4} = 1$$

You may need to reset the start value and incremental values for the table using

2nd TBLSET

X	Y1
0	1
1	4
2	6
3	4
4	1
5	0

press + for $\Delta|b|$

5.12 Calculating binomial probabilities

Example 41

X is a discrete random variable and $X \sim \text{Bin}(9, 0.75)$.

Calculate $P(X = 5)$

$$P(X = 5) = \binom{9}{5} 0.75^5 0.25^4$$

The calculator can find this value directly.

Press $\text{2nd DISTR A:binompdf(}$

Enter 9 as trials, 0.75 as p and 5 as x .

Select Paste and press ENTER

Press ENTER again

You should enter the values: n (numtrials), p and x , in order.

```

DISTR DRAW
0:pdfcdf(
1:binompdf(
2:binomcdf(
3:poissonpdf(
4:poissoncdf(
5:geometpdf(
6:geometcdf(
  
```

The calculator shows that $P(X = 5) = 0.117$ (to 3 sf).

```

binompdf(9,0.75)
.1167984009
  
```

Example 42

X is a discrete random variable and $X \sim \text{Bin}(7, 0.3)$.

Calculate the probabilities that X takes the values $\{0, 1, 2, 3, 4, 5, 6, 7\}$.

Press **2nd** **DISTR** A:binompdf(.

Enter 7 as trials, 0.3 as p and leave x blank.

Select Paste and press **ENTER**

Press **ENTER** again

You should enter
the values: n (numtrials), p
and x , in order.

```

DRAW
0:pdfcdf(
1:binompdf(
2:binomcdf(
C:Poissonpdf(
D:Poissoncdf(
E:geometpdf(
F:geometcdf(

```

The calculator displays each of the probabilities.

To see the remaining values scroll the screen to the right.

The list can also be transferred as a list.

```

binompdf(7,0.3)
(.0823543 .24701)

```

Press **STO** **2nd** **L1**.

Press **ENTER**.

```

binompdf(7,0.3)
(.0823543 .24701)
Ans→L1
(.0823543 .24701)

```

Press **STAT** 1:Edit...

The binomial probabilities are now displayed in the first column.

L1	L2	L3	1
.0823543	-----	-----	
.24701			
.31765			
.22689			
.09724			
.025			
.00357			
L1(1) = .0823543			

Example 43

X is a discrete random variable and $X \sim \text{Bin}(20, 0.45)$.

Calculate

- the probability that X is less than or equal to 10.
- the probability that X lies between 5 and 15 inclusive.
- the probability that X is greater than 11.

Press **2nd** **DISTR** B:binomcdf(.

You are given the lower
bound probability so you
have to calculate other
probabilities using this.

You should enter
the values: n (numtrials),
 p and x , in order.

```

DRAW
0:pdfcdf(
A:binompdf(
1:binomcdf(
C:Poissonpdf(
D:Poissoncdf(
E:geometpdf(
F:geometcdf(

```

▶ Continued on next page

<p>a Enter 30 as trials, 0.45 as p and 10 as x. Select Paste and press ENTER Press ENTER again $P(X \leq 10) = 0.751$ (to 3sf).</p>	<pre>binomcdf(20,0.4,10) .75071064</pre>
<p>b $P(5 \leq X \leq 15) = P(X \leq 15) - P(X \leq 4)$ Press 2nd DISTR B:binomcdf(Enter 20 as trials, 0.45 as p and 10 as x. Select Paste and press ENTER Type (-) and then Press 2nd DISTR B:binomcdf(Enter 20 as trials, 0.45 as p and 4 as x. Select Paste and press ENTER Press ENTER again $P(5 \leq X \leq 15) = 0.980$ (to 3sf).</p>	<pre>binomcdf(20,0.4,15) .9796059841</pre>
<p>c $P(X > 11) = 1 - P(X \leq 11)$ Enter 1 (-) and then Press 2nd DISTR B:binomcdf(Select Paste and press ENTER Press ENTER again $P(X > 11) = 0.131$ (to 3sf).</p>	<pre>1-binomcdf(20,0.4,11) .130764971</pre>

Calculating normal probabilities

5.13 Calculating normal probabilities from X -values

Example 44

A random variable X is normally distributed with a mean of 195 and a standard deviation of 20 or $X \sim N(195, 20^2)$. Calculate

- a** the probability that X is less than 190.
- b** the probability that X is greater than 194.
- c** the probability that X lies between 187 and 196.

Press **2nd** **DISTR** | 2:normalcdf(.

Press **ENTER**.

You should enter the values, Lower Bound, Upper Bound, μ and σ , in order.

The value E99 is the largest value that can be entered in the GDC and is used in the place of ∞ . It stands for 1×10^{99} ($-E99$ is the smallest value and is used in the place of $-\infty$). To enter the E, you need to press **2nd** **EE**.

```
DISTR DRAW
1:normalpdf(
2:normalcdf(
3:invNorm(
4:invT(
5:tPdf(
6:tcdf(
7:χ²pdf(
```

- a** $P(X < 190)$
 Enter Lower Bound as $-E99$, Upper Bound as 190, μ to 195 and σ to 20.
 $P(X < 190) = 0.401$ (to 3sf)

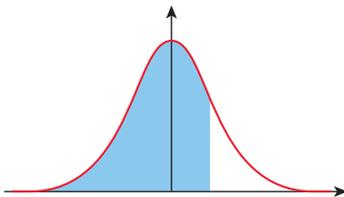
```
normalcdf(-E99,
199,190,195,20)
normalcdf(-E99,
.4012937256
```

▶ Continued on next page

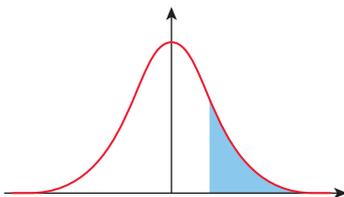
<p>b $P(X < 194)$ Enter Lower Bound as 194, Upper Bound as E99, μ as 195 and σ as 20. $P(X > 194) = 0.520$ (to 3 sf)</p>	<pre>normalcdf(194, E99, 194, E99, 195, 20) normalcdf(194, E99, .519938874</pre>
<p>c $P(187 < X < 196)$ Enter Lower Bound as 187, Upper Bound as 196, μ as 195 and σ as 20. $P(187 < X < 196) = 0.175$ (to 3 sf)</p>	<pre>normalcdf(187, 196, 187, 196, 195, 20) normalcdf(187, 196, .1753605711</pre>

5.14 Calculating X-values from normal probabilities

In some problems you are given probabilities and have to calculate the associated values of X . To do this, use the `invNorm` function.



When using the Inverse Normal function, make sure you find the probability on the correct side of the normal curve. The areas are always the lower tail, that is they are always in the form $P(X < x)$ (see Example 26).



If you are given the upper tail $P(X > x)$, you must first subtract the probability from 1 before you can use `invNorm` (see example 27).

Example 45

A random variable X is normally distributed with a mean of 75 and a standard deviation of 12 or $X \sim N(75, 12^2)$.
 If $P(X < x) = 0.4$, find the value of x .

You are given a lower-tail probability so you can find $P(X < x)$ directly.

Press `2nd` `DISTR` | 3:invNorm(.

You should enter the values: area (probability), μ and σ , in order.

```
0:stat
1:normalpdf(
2:normalcdf(
3:invNorm(
4:invT(
5:tpdf(
6:tcdf(
7:χ²pdf(
```

Press `ENTER`.

Enter area (probability) as 0.4, μ as 75 and σ as 12.
 So if $P(X < x) = 0.4$ then $x = 72.0$ (to 3 sf).

```
invNorm(0.4, 75, 12)
71.95983479
```

Example 46

A random variable X is normally distributed with a mean of 75 and a standard deviation of 12 or $X \sim N(75, 12^2)$. If $P(X > x) = 0.2$, find the value of x .

You are given an upper-tail probability so you must first find $P(X < x) = 1 - 0.2 = 0.8$. You can now use the invNorm function as before.

Press **2nd** **DISTR** | 3:invNorm(.

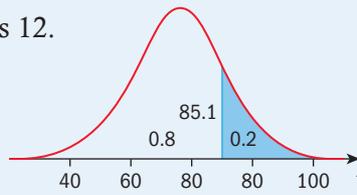
Press **ENTER**.

You should enter the values: area (probability), μ and σ , in order.

```

DISTR DRAW
1:normalpdf(
2:normalcdf(
3:invNorm(
4:invT(
5:tpdf(
6:tcdf(
7:χ²pdf(
    
```

Enter area (probability) as 0.8, μ as 75 and σ as 12. So if $P(X > x) = 0.2$ then $x = 85.1$ (to 3 sf).



▲ This sketch of a normal distribution curve shows this value and the probabilities from Example 29.

```

invNorm(0.8,75,12
85.0994548
    
```

Scatter diagrams, linear regression and the correlation coefficient

5.15 Scatter diagrams

Example 47

Consider this data that is approximately connected by a linear function.

x	1.0	2.1	2.4	3.7	5.0
y	4.0	5.6	9.8	10.6	14.7

- Find the equation of the least squares regression line of y on x .
- Find Pearson's product-moment correlation coefficient.
- Use the equation to predict the value of y when $x = 3.0$.

Press **STAT** | 1:Edit and press **ENTER**.

Type the values of x in the first column (L1) and the values of y in the second column (L2).

Press **ENTER** or **↓** after each number to move down to the next cell.

Press **→** to move to the next column.

You can use columns from L1 to L6 to enter the lists.

```

STAT CALC TESTS
1:Edit
2:SortA(
3:SortD(
4:ClrList
5:SetUpEditor
    
```

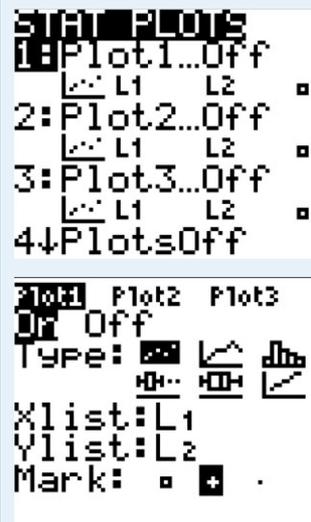
L1	L2	L3	2
1	4	-----	
2.1	5.6		
2.4	9.8		
3.7	10.6		
5	14.7		
-----	-----		

L2(6) =

▶ Continued on next page

Press **2nd** **STAT PLOT** and **ENTER** to select Plot1.

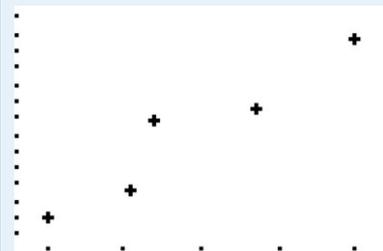
Select On, choose the scatter diagram option, XList as L1 and Ylist as L2. You can choose one of the three types of mark.



Press **ZOOM** | 9:Stat.

The automatic scales do not usually give the best display of the scatter diagram. You will need to change the default values.

You may need to delete any function graphs. **Y=**



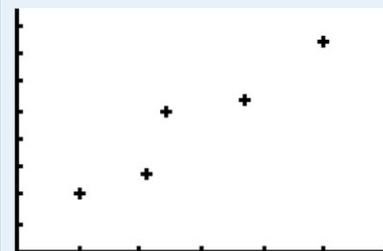
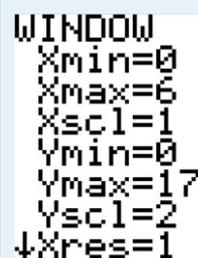
Press **WINDOW** and choose options as shown.

Xmin and Xmax should include the range of the x -data.

Ymin and Ymax should include the range of the y -data.

You now have a scatter graph of y against x .

You need to include zero in the range if you want to show the axes on the graph.



▶ Continued on next page

In order to see the Pearson's product-moment correlation coefficient, you need to have diagnostics on your GDC switched on.

Press **MODE** and use **▼** to scroll down to the second screen. Set STAT DIAGNOSTICS to ON and press **ENTER**.

Then press **2nd** **QUIT** to return to the home screen.

```

      †BACK†
MATHPRINT CLASSIC
2nd Un/d
ANSWERS: AUTO DEC FRAC
GOTO FORMAT GRAPH: 00 YES
STAT DIAGNOSTICS: OFF 00
SET CLOCK 08/14/11 6:09PM

```

```
LinReg(ax+b) ■
```

Press **STAT** | **CALC** | 4:LinReg($ax + b$).

Press **2nd** **L1** , **2nd** **L2** , .

Press **ALPHA** **F4** and press **ENTER** to select Y1.

Press **ENTER** again.

```
4:LinReg(ax+b) L1,L2,
```

```

Y1 Y6
Y2 Y7
Y3 Y8
Y4 Y9
Y5 Y0

```

```
FRAC FUNC MTRX VAR
```

You will see the coefficients of the equation of the least squares regression line and the value r of the correlation coefficient.

The equation is $y = 2.63x + 1.48$ (to 3 sf).

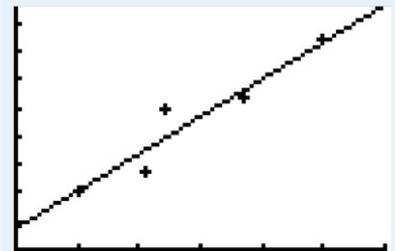
The value of r is 0.955 (to 3 sf).

```

LinReg
y=ax+b
a=2.628199748
b=1.475912715
r²=.9115303479
r=.9547409847

```

Press **GRAPH** and you will see the least squares regression line and the data points that you plotted previously.



▶ Continued on next page

Press **TRACE** and use the **▶** **◀** keys to move the trace along the line.

The cursor moves between the data points.

Press **▲** to move onto the line itself.

It is not possible to move the trace point to an exact value, so get as close to $x = 3$ as you can.

From the graph, you have found that y is approximately 9.5 when $x = 3.0$.

Press 3 **ENTER**.

The cursor now moves to exactly 3.0.

When $x = 3.0$, an estimate of the value of y is 9.36, from the graph.

